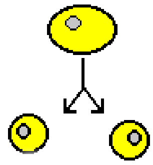
Name: Class: Date:

1-Doubling rule



1. Using the iteration rule **multiply by 2**, write the first ten numbers in the orbits for the following seeds:

Seed = 1 **Orbit:**

Seed = 1.5 Orbit:

Seed = 0 Orbit:

Seed = − 1 Orbit:

2. Explain the **eventual outcomes** for these various seeds. The boxes below will help you.

-
-
-
-

The orbit of …… tends to positive infinity	The orbit of …… tends to negative infinity
The orbit of …… tends to zero	The orbit of …… stays fixed

3. **Predict** the eventual outcomes using the seeds 6 and − 6. Use the boxes above and the future …**will** … .

-
-

4. Did all the seeds produce the same behaviour? Explain.

- **If** the seed is
-
-

Name: Class: Date:

2-Halving rule

1. Using the iteration rule **divide by 2**, write the first eight numbers in the orbits for the following seeds:

Seed = 10 **Orbit:**

Seed = 0 Orbit:

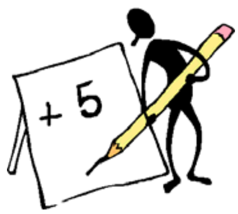
Seed = − 4 Orbit:

2. What is the **eventual outcome** of the iteration in each case?

-
-
-

3. Did all the seeds produce the same behaviour? Explain.

3-Adding five



1. Using the iteration rule **add 5**, write the first nine numbers in the orbits for the following seeds:

Seed = 40 **Orbit:**

Seed = 0 Orbit:

Seed = − 400 Orbit:

2. Comment on the **fates** of the orbits.

Name: Class: Date:

4-Sign changing

1. Using the iteration rule **sign changing**, write the first ten numbers in the orbits for the following seeds:

Seed = 3 **Orbit:**

Seed = − 4 Orbit:

Seed = 0 Orbit:

2. We call the orbits of the seeds 3 and − 4 **cycles of period 2.** Why?

3. What can you say about the orbit of the seed 0?

5-Arrow notation

Write each rule using arrow notation and **check it** with your partner:

Rule	**Arrow notation**
Add three	$x \rightarrow x + 3$
Divide by four, then add two	$x \rightarrow \frac{x}{4} + 2$
Multiply by minus five	
Take the opposite, then add one	
Subtract seven point five	
Subtract six, then multiply by four	
Subtract six, then divide by two	
Multiply by three point five	
Add two thirds	

Name: Class: Date:

6-Some other rules

For each of the following iteration rules, write the first ten numbers in the seed's orbits, and describe the fate of the orbit.

1. Rule: $x \rightarrow \frac{x}{5}$ Seed = 5000

   Orbit:

   Fate:

2. Rule: $x \rightarrow 5x$ Seed = 0.5

   Orbit:

   Fate:

3. Rule: $x \rightarrow 3x + 6$ Seed = -3

   Orbit:

   Fate:

4. Rule: $x \rightarrow -x + 1$ Seed = 1

   Orbit:

   Fate:

Work in pairs:

1. Find one iteration rule that the orbit of all seeds, except 0, is approaching 0.

2. Find one iteration rule that works for the orbit: $1 \rightarrow 3 \rightarrow 7 \rightarrow 15 \rightarrow 31 \rightarrow 63 \rightarrow 127 \rightarrow 255 \rightarrow ...$

3. Find one iteration rule that works for the orbit: $1 \rightarrow 2 \rightarrow 5 \rightarrow 14 \rightarrow 41 \rightarrow 122 \rightarrow 365 \rightarrow$ ...

Name: Class: Date:

7-Squaring rule

1. Using the **squaring** iteration rule, $x \to x^2$, write the first six numbers in the orbits for the following seeds:

Seed = 1 Orbit:

Seed = − 1 Orbit:

Seed = 0 Orbit:

Seed = 2 Orbit:

Seed = − 2 Orbit:

Seed = 3 Orbit:

Seed =10 Orbit:

Seed = − 0.1 Orbit:

Seed = − 0.5 Orbit:

Seed = 0.5 Orbit:

Seed = 0.2 Orbit:

2. **Discuss** with your partner what the fate of these orbits is and **classify** the seeds above according to the same behaviour.

3. **Predict** what will happen to the orbit of **any** seed under this iteration rule. The word bank below will help you.

Greater than	between	equal to	less than

Name: Class: Date:

8-Square root rule

1. Use a calculator to write the first six numbers in the orbits (rounded to two decimal places) for the following seeds under the **square root** iteration rule, $x \to \sqrt{x}$:

Seed = 1 Orbit:

Seed = 0 Orbit:

Seed = 16 Orbit:

Seed = 10 000 Orbit:

Seed = 0.1 Orbit:

Seed = 0.9 Orbit:

2. What do you think will happen to **any positive** seed under this iteration rule?

9-Cubing rule

1. Using the **cubing** iteration rule, $x \to x^3$, write the first five numbers in the orbits for the following seeds:

Seed = 1 Orbit:

Seed = − 1 Orbit:

Seed = 0 Orbit:

Seed = 2 Orbit:

Seed = − 2 Orbit:

Seed = 0.1 Orbit:

Seed = − 0.1 Orbit:

Seed = − 0.5 Orbit:

Seed = 0.5 Orbit:

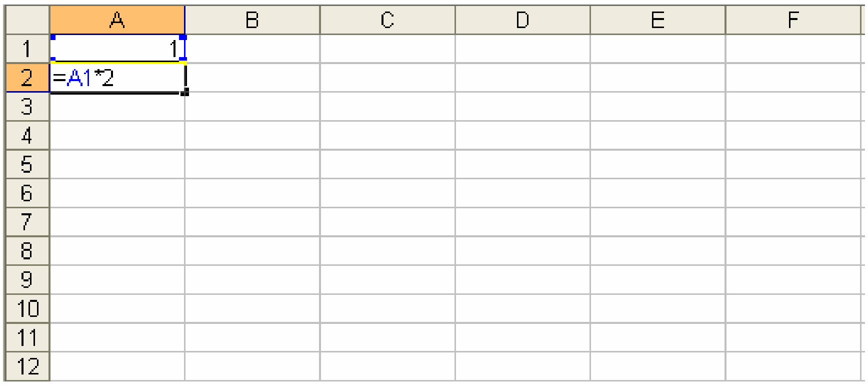
2. **Predict** what will happen to the orbit of **any** seed under this iteration rule.

Name: Class: Date:

10-Spreadsheets

Use a computer to check the previous exercises:

1. - Open a spreadsheet and enter the seed 1 into the first cell A1.
   - Enter the doubling iteration rule, given by the formula = **A1*2**, into the second cell A2.



   - **Dragging** this cell down the column you create the orbit of the seed 1 under the doubling rule.

   - Enter the seed 1.5 into the first cell of the second column B1.
   - Drag the cell A2 to the cell B2.
   - Dragging B2 down you create in the second column B the orbit of the seed 1.5.

   - Repeat this process entering the seeds 0, − 1, 6, − 6.

Observe the fates of these six orbits and **check** your previous answer.

2. Open a new spreadsheet and proceed as above to check the exercise 2-Halving rule using the formula =**A1/2**.

3. Open a new spreadsheet and check the exercise 3-Adding five using the formula =**A1+5**.

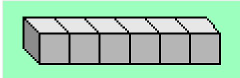
4. Open a new spreadsheet and check the exercise 7-Squaring rule using the formula =**A1^2**.

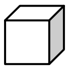
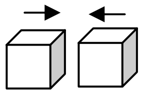
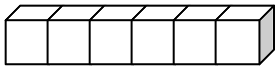
5. Open a new spreadsheet and check the exercise 8-Square root rule using the formula =**SQRT(A1)**.

6. Open a new spreadsheet and check the exercise 9-cubing rule using the formula =**A1^3**.

Name: Class: Date:

11-Counting edges



1. How many **edges** does a **cube** have? 
2. If you **glue two** cubes as at the picture, how many edges do you **add**? 
3. Glue a third cube. How many edges does the shape have?
4. How many edges does the figure at the picture have? 
5. Notice that you have a **numerical iteration** going on. Determine **with your partner**:

   Iteration rule:
   Seed:
   Orbit:

6. Calculate **in pairs** how many edges a similar structure made up of 100 cubes have. Explain your strategy.

12-Counting faces

This time focus on counting the **square faces** on the **outside** of each structure.

1. Write down by yourself the six questions of the exercise as above in a blank sheet.
2. Answer them as before.

Name: Class: Date:

13-Appreciation

1. A house valued at six hundred and fifty thousand euros **appreciates** for three years by 7% every year. Calculate the value of the house after those three years.

Original amount = **100%** **New** amount = **107%**

First year appreciation: 107 % of 650 000 = 1.07 times 650 000 = 695 500

Second year appreciation: 107 % of 695 500 =

Third year appreciation: 107 % of

Notice that you have a **numerical iteration** going on. Determine **with your partner**:

Iteration rule:
Seed:
Orbit:

2. A Miquel Barceló painting, bought for 6 000 euros, appreciated by 150% per year during three years. What was the painting worth after three years?

Homework: Find information about Miquel Barceló in the Internet.

3. The pressure in a boiler is 180 pascals. A faulty valve causes the pressure to **rise** in the boiler by 12% every hour. The situation will become dangerous when the pressure reaches 300 pascals. If it continues to rise this way, during which hour will the boiler's pressure reach danger level?

Name: Class: Date:

14-Depreciation

1. A car is originally worth 12 900 euros. It **depreciates** over three years by 7% every year. Calculate its worth after those three years.

Original amount = **100%** **New** amount = **93%**

First year depreciation: 93 % of 12 900 = 0.93 times 12 900 =

Second year depreciation: 93 % of

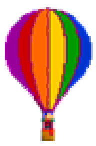
Third year depreciation:

Notice that you have a **numerical iteration** going on. Determine **with your partner**:

Iteration rule:
Seed:
Orbit:

2. Peter bought his first motorbike at the start of 2005 for 9 400 euros. It **depreciated** in value by 15% every year he owned it. What was the bike worth by the end of 2007?

3. A hot air balloon was drifting along at a height of 5 000 m when it developed a leak. The balloon **dropped** by 8% every minute after that. What was the balloon's height after three minutes?



Name: Class: Date:

15-Taking medication

After you swallow a 100-units capsule, the medication is slowly absorbed by your body. Suppose 25% of the amount in your system is **eliminated** every hour.
- How much of the medication **remains** in your system after 5 hours?

Original amount = **100%** **New** amount = **75%**

After 1 hour: 75 % of 100 = 0.75 times 100 = 75 units

After 2 hours: 75 % of 75 = 0.75 times 75 =

After 3 hours:

After 4 hours:

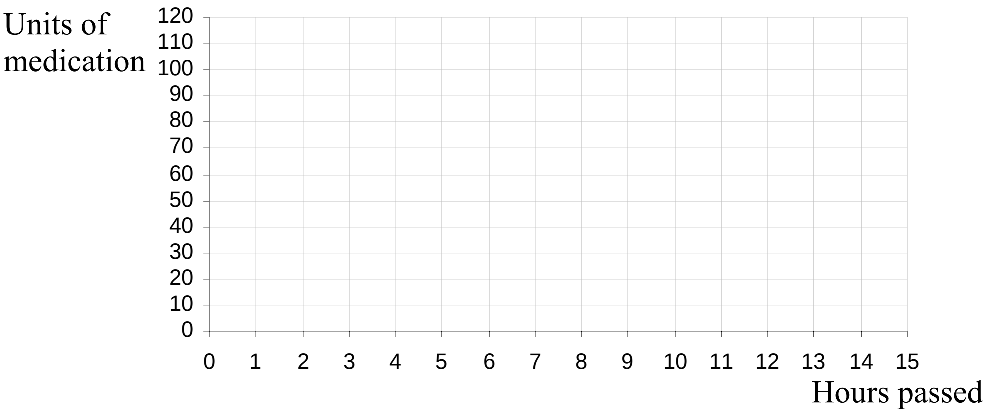
After 5 hours:

- Notice that you have a **numerical iteration** that describes the amount of medication in your body. Determine **with your partner**:

Iteration rule:
Seed:
Orbit:

- Use a spreadsheet or calculator to compute the amount of medication in your body the next 10 hours.

- Sketch a plot of these 15 points.

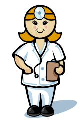


- What is the fate of this orbit?

Name: Class: Date:

16-Daily doses

Your doctor prescribes you the following medication: “Take an initial dose of 75 units and 25 units every day after”. It is known that your body eliminates 20% of the medication every day.



- How much is the amount of medication in your system after 4 days?

Original amount **= 100%** **New** amount **= 80%**

After 1 day: 80 % of 75 = 0.8 times 75 = 60 units
60 + 25 = 85 units

After 2 days: 80 % of 85 =

After 3 days:

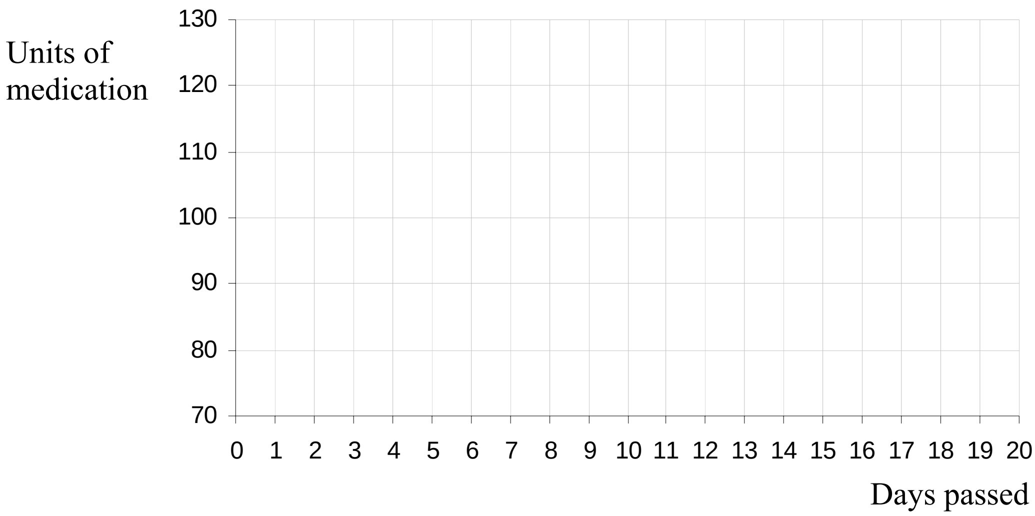
After 4 days:

- Notice that you have a **numerical iteration** that describes the amount of medication in your body. Determine **with your partner**:

Iteration rule:
Seed:
Orbit:

- If the **toxic level** in your body is 130 units, is it **safe** for you to continue taking this medication? Use a spreadsheet or calculator and **argue** with your partner.

- Sketch a plot of yours answers.



Name: Class: Date:

17-Savings account

Special Savings Rate 4.2 % p.a.

1. A bank is paying 4.2% **interest** per year on your savings account, and the interest is paid once per year at the end of the year.
   You have 800 euros to deposit in the account.
   You want to know how much money you will have at the end of **each** year over a five-year period.

Original amount **= 100%** **New** amount **= 104.2%**

First year balance: 104.2 % of 800 = 1.042 times 800 =

Second year balance: 104.2 % of

Third year balance:

Fourth year balance:

Fifth year balance:

Notice that you have a **numerical iteration** going on. Determine **with your partner**:

Iteration rule:
Seed:
Orbit:

Use a spreadsheet or calculator to find the balance in your account after ten years.

2. Joan was told that if she left her savings of 2400 euros in the Scotia Bank for 5 years they would give her a special annual **rate** of 5.4%. How much would her 2400 euros be worth at the end of the 5 year period?

Name: Class: Date:

18-Savings plan

Suppose that the interest rate in a savings account is 5% annually and that your initial deposit is 4500 euros. If you add 500 euros to this account at the end of each year, what is your balance at the end of the first 6 years?

Original amount **= 100%** **New** amount **=105%**

First year balance: 105 % of 4500 = 1.05 times 4500 = 4725
+ 500
5225

Second year balance: 105 % of 5225 =

Third year balance:

Fourth year balance:

Fifth year balance:

Sixth year balance:

Notice that you have a **numerical iteration** going on. Determine **with your partner**:

Iteration rule:
Seed:
Orbit:

Use a spreadsheet or calculator to find the balance in your account after ten years.

Name: Class: Date:

19-Doubling your investment

1. An internet bank offers a tremendous interest rate of 9% per year on savings. If you invest your savings of 6000 euros in the internet bank:

    a) How much will be your savings worth after 1 year?

    b) How much will be your savings worth after 2 years?

    c) How much will be your savings worth after 3 years?

    d) What iteration rule computes account balances using this interest rate?

    e) How many years does it take for you to have more than 12 000 euros in the bank?

    f) In how many years will you have 15 000 euros in the bank?

2. The Scotia Bank offers its costumers an interest rate of 4.5% p.a. on their savings. Ted deposits 1200 euros in a new Scotia bank account. How many years will it take to **double** his investment?

Name: Class: Date:

20-Loans

Mr. Brown **borrows** 12 000 euros **from** a bank to buy a car. The interest rate is 1% per month and he makes monthly payments of 300 euros.

- List his loan balances after the first 5 months.

Original amount **= 100%** **New** amount **= 101%**

Initial balance: −12 000

After 1 month: 101 % of −12 000 = 1.01 times −12 000 = − 12 120

− 12 120 + 300 = − 11 820

After 2 months: 101 % of −11 820 = 1.01 times −11 820 = −

+ 300 =

After 3 months:

After 4 months:

After 5 months:

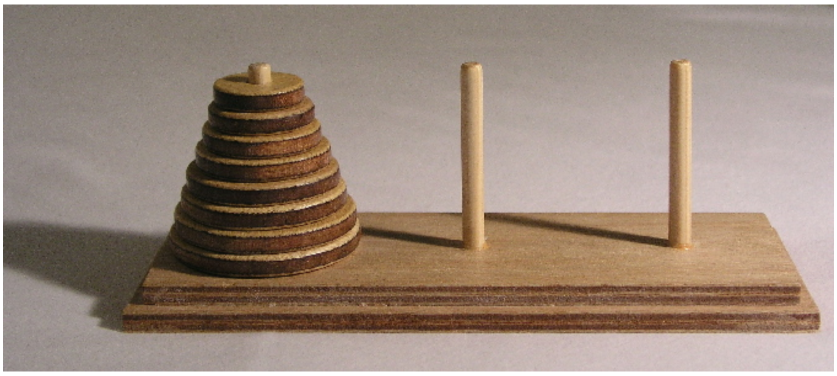
- Notice that you have a **numerical iteration** going on. Determine **with your partner**:

Iteration rule:
Seed:

- Use a spreadsheet to find the time needed to **pay back** the loan.

Name: Class: Date:

21-The tower of Hanoi



A model set of the Tower of Hanoi (with 8 disks)

1. The game
It consists of three pegs, and a number of disks, each one larger than the one above. The aim is to move all the disks onto another peg, in the **smallest** possible number of moves. You must follow these rules:

- You can only move one disk at a time.
- You can never put a larger disk on top of a smaller disk.

2. How to play?
Provide you with disks of different sizes. For example, draw circles of increasing radius on cardboard or even regular paper, and cut them out.
The pieces can be stacked on top of each other to simulate the game.

The **best strategy** is to start with simple versions and look for patterns.

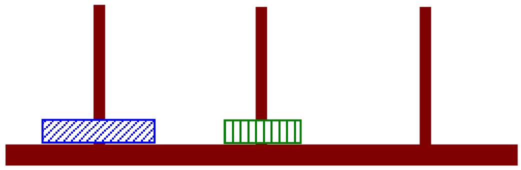
3. Only one disk

You need only **one** move.

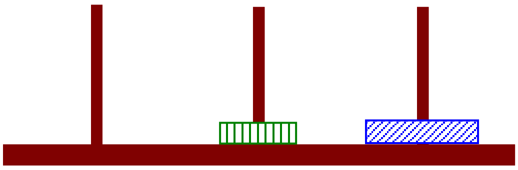
Name: Class: Date:

4. Two disks

original puzzle



move disk **1**



move disk **2**

move disk **1**

At least **three** moves are required. We can describe the solution with the sequence of numbers **1 2 1** (it gives the order in which the disks must move).

5. Three disks

Original puzzle

- How many moves do you need to move the three disks onto another peg?

- Describe your solution by giving the sequence of disks numbered 1, 2 and 3, from the top to the bottom.

- Check your answer with your **partners**.

- **Notice** that in the process of solving this puzzle, you **first** solve the two-disk puzzle, **then** move the bottom disk to an empty peg and **finally** solve the puzzle for two disks again.

Name: Class: Date:

6. Four disks

First make sure you understand how to solve the puzzle when given only one disk, two disks, or three disks to start with.
Now solve the puzzle for four disks.
Describe the sequence of disks that you must move using 1, 2, 3 and 4, as before.

7. The game on the Web

Run simulations of the Tower of Hanoi puzzle, for example at

http://www.mathsnet.net/puzzles/hanoi/index.html

8. Five disks

Describe your strategy for completing the five-disk puzzle. Try to use the word **iteration.**

9. Solving the game

List the total number of moves necessary to solve the puzzle for 1 disk, for 2 disks, for 3 disks, for 4 disks and for 5 disks.

Regard this is an orbit generated by numerical iteration. Find out:

- Seed
- Iteration rule

How many moves do you need to solve the puzzle with 6 disks? **Discuss** with your partner and **explain** how you found this out.

Name: Class: Date:

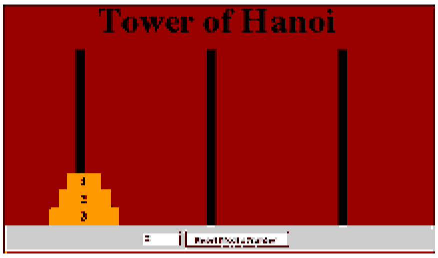
10. Read the text and **answer** the questions below:

Fascinating Facts


Temple Pura Ulu Danau, Bali

The Tower of Hanoi (sometimes referred to as the Tower of Brahma or the End of the World Puzzle) was invented by the French mathematician, Edouard Lucas, in 1883. He was inspired by a legend that tells of a Hindu temple where the pyramid puzzle might have been used for the mental discipline of young priests. Legend says that at the beginning of time the priests in the temple were given a stack of 64 gold disks, each one a little smaller than the one beneath it. Their assignment was to transfer the 64 disks from one of the three poles to another, with one important proviso: a large disk could never be placed on top of a smaller one. The priests worked very efficiently, day and night. When they finished their work, the myth said, the temple would crumble into dust and the world would vanish.

Where's the Math in this Game?



The number of separate transfers of single disks the priests must make to transfer the tower is 2 to the 64th minus 1, or 18,446,744,073,709,551,615 moves! If the priests worked day and night, making one move every second it would take slightly more than 580 billion years to accomplish the job! You have a great deal fewer disks than 64 here. Can you calculate the number of moves it will take you to move the disks from one of the three poles to another?

http://www.lhs.berkeley.edu/Java/Tower/tower.html

- Who invented The Tower of Hanoi puzzle?

- When was it invented?

- What was he inspired by?

- How many years are a billion of years?

- According to the text the priests need $\mathbf{2^{64} - 1}$ moves to transfer **64** disks from one pole to another. **Check** this formula with yours previous solutions of the puzzle. **Argue** about it with your partners and your teacher

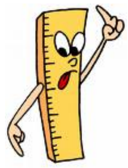
Name: Class: Date:

1-Shrinking iteration rules

1. The seed is the **straight line segment** below. The iteration rule is to **shrink** the segment so that its length is **half** the length of the original one.

→ →

a) **Draw** above the third segment of the orbit. Use your **ruler**.



b) Assume the **length** of the original straight line segment is **1 unit**.
-What **fraction** is the length of the second segment?

-What fraction is the length of the third segment?

c) Draw below the next three segments along the orbit.

d) What fractions are the lengths of these three segments?

e) In the grid below **tick** true or false beside each statement.

	True	False
The fate of the orbit is a single point.		
The fate of the orbit is a very small segment.		
The orbit tends to zero.		
The orbit disappears.		
The length of the orbit's segments tends to zero.		

f) Check your answers with your partner.

Name: Class: Date:

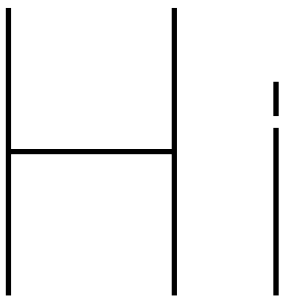
2. a) **Sketch** the next three pictures in the orbit using the iteration rule: “Shrink the previous figure so that each side is half as **long** and half as **wide**”.
Use your ruler and your **set square**.

 →

b) Is the orbit approaching a single point or two points?

c) Why?

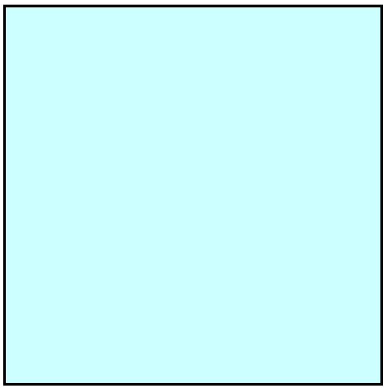
3. The seed is the word “Hi” and the iteration rule is: **Shrink** the word “Hi” by a **scale factor of** $\frac{1}{2}$.

 →

a) Sketch the next 3 words in the orbit.
b) Explain the fate of the orbit.

Name: Class: Date:

4. The seed is the square below. The iteration rule is to shrink the square so that each **side** is half as long.



→

a) Draw the next four squares of the orbit.
b) What is the fate of the orbit?

c) - Assume the length of each side of the original square is **1 unit**.
- Remember:
- *The **perimeter** of a **square** is the total length of the four sides.*
- *The **area** of a **square** can be found by multiplying the length of two sides.*

- Calculate the length of each side of these four squares, their perimeter and their area and fill in the grid below:

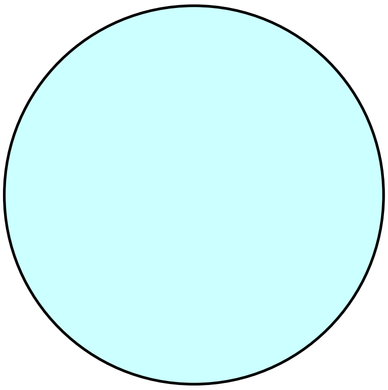
	Side length (as a decimal)	Side length (as a fraction)	Perimeter (in units)	Area (in square units)
Original square	**1**	**1**	$\mathbf{4 \cdot 1 = 4}$	$\mathbf{1^2 = 1}$
First iteration	**0.5**	$\frac{1}{2}$	$4 \cdot \frac{1}{2} = \frac{4}{2} = 2$	$\left(\frac{1}{2}\right)^2 = \frac{1}{2^2} = \frac{1}{4}$
Second iteration				
Third iteration				
Fourth iteration				

d) Complete the sentences:
- The side length is decreasing by a scale factor of.........................
- The perimeter is decreasing by a scale factor of.........................
- The area is decreasing by a scale factor of...........................

e) Predict (as a fraction) the side length, the perimeter and the area of the eleventh iteration.

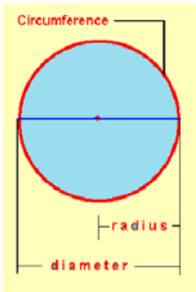
Name: Class: Date:

5. The seed is the **circle** below and the iteration rule is to shrink the circle so that the **diameter** is half as long.

 →

a) Draw the next four circles of the orbit.
b) What is the fate of the orbit?

c) - Assume the diameter's length of the original circle is **2 units**.
- Remember:
Circumference $= 2 \cdot \pi \cdot radius = \pi \cdot diameter$
Circle Area $= \pi \cdot r^2$



- Complete the table below:

	Diameter	Radius	Perimeter (in units)	Area (in square units)
Original circle	**2**	**1**	$\boldsymbol{\pi \cdot 2 = 2\pi}$	$\boldsymbol{\pi \cdot 1^2 = \pi \cdot 1 = \pi}$
First iteration	**1**	$\frac{1}{2}$	$\boldsymbol{\pi \cdot 1 = \pi}$	$\pi \cdot \left(\frac{1}{2}\right)^2 = \pi \cdot \frac{1}{2^2} = \frac{\pi}{4}$
Second iteration				
Third iteration				
Fourth iteration				

d) Complete the sentences:
- The diameter and the radius are decreasing by a scale factor of...................
- The perimeter is decreasing by a scale factor of.........................
- The area is decreasing by a scale factor of...........................

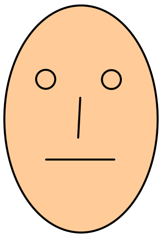
e) Predict (as a fraction) the diameter, the radius, the perimeter and the area of the eleventh iteration.

Name: Class: Date:

2-Rotating iteration rules

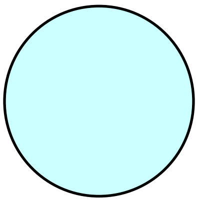
1. Using the iteration rule "**Rotate the figure 90º clockwise**", draw down the next four pictures in the orbits for the following seeds:

Ellipse with a face



→

Circle



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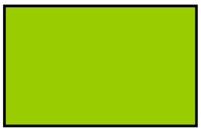
Rectangle



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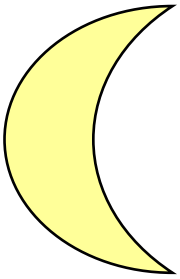
Waning Moon



→

2. Describe the fate of these orbits.

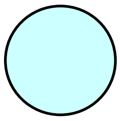
-
-
-
-

Name: Class: Date:

3-Replacement iteration rules

1. a) Using the seed "the circle below", draw down the next three pictures in the orbits for the following iteration rules:

Place a circle of the **same size** just to the right of the previous circle



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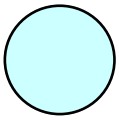
Place an exact copy of the **previous figure** just to the right of the previous figure



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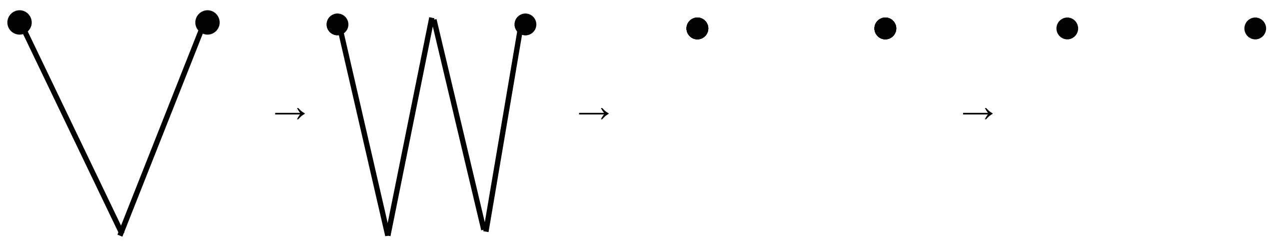
→

b) How many circles you will have in the tenth iteration?

First rule: .... Second rule: ....

c) **Describe** the fate of these orbits.

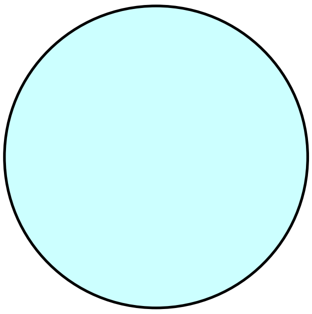
2. a) Follow the iteration:



b) What **shape** is the fate of this orbit?

Name: Class: Date:

3. The seed is the **circle** below and the iteration rule is to **replace** a circle by **two smaller copies** of itself (lined up side by side) whose diameters are each one half of the original



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a) Draw the next three pictures in the orbit.

b) How many circles you will have in the ninth iteration?

c) What shape is the orbit approaching?

d) - Assume the diameter's length is **2 units**.

- Complete the table below:

	Perimeter (in units)	Area (in square units)
Original circle	$\boldsymbol{\pi \cdot 2 = 2\,\pi}$	$\boldsymbol{\pi \cdot 1^2 = \pi \cdot 1 = \pi}$
First iteration	$\boldsymbol{2 \cdot \pi \cdot 1 = 2\,\pi}$	$2 \cdot \pi \cdot \left(\frac{1}{2}\right)^2 = 2 \cdot \pi \cdot \frac{1}{2^2} = \frac{\pi}{2}$
Second iteration		
Third iteration		

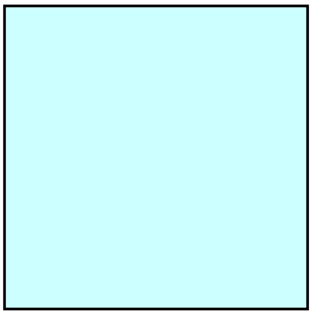
e) Predict the perimeter and the area of the ninth iteration.

Name: Class: Date:

4-Midpoint iteration rules

1. The seed is the **filled** square below. The iteration rule is:
   - Sketch the **boundary** of the previous square.
   - Find the **midpoint** of each side.
   - Connect **adjacent** midpoints with a straight line segment to form a new square.
   - **Remove** the old square and **shade** the interior of the new square.

a) Draw the next three pictures in the orbit.



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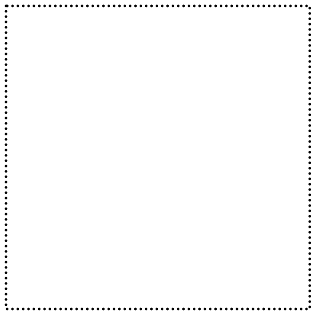


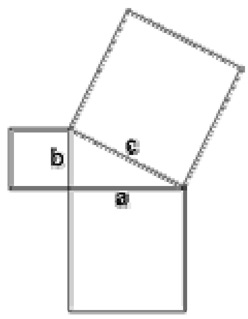
b) Express the area of these three squares as a percentage of the area of the original square.

c) What is the fate of the orbit?

d) Assume the length of each side of the original square is **2 units.**

Remember

Pythagoras' Theorem:
In a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.



- Calculate the side length of the first iteration.

- Calculate the side length of the second iteration.

- Calculate the side length of the third iteration.

- Complete the table below.

	Side length (in units)	Perimeter (in units)	Area (in square units)
Original square	**2**		
First iteration	$\sqrt{2}$		
Second iteration			
Third iteration			

- Compare the areas founded to your answer b)

e) Predict the area of the sixth iteration.

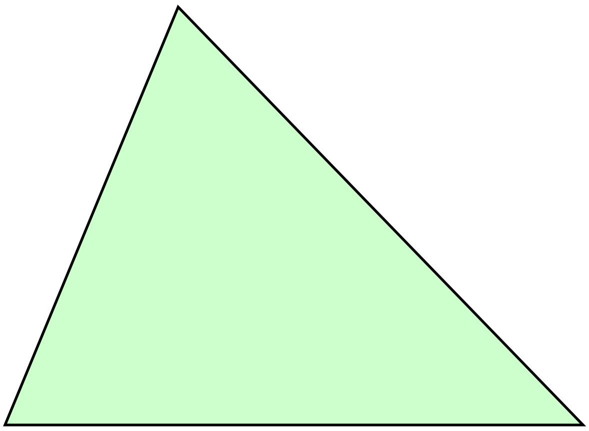
f) Complete the sentences:
- The area is decreasing by a scale factor of..........................
- The side length is decreasing by a scale factor of...........................

g) Predict the side length of the sixth iteration.

Name: Class: Date:

2. The seed is **any** triangle and the iteration rule "**Join** the midpoints of each of the sides of the triangle to form a new triangle".

a) Draw the next three pictures in the orbit.



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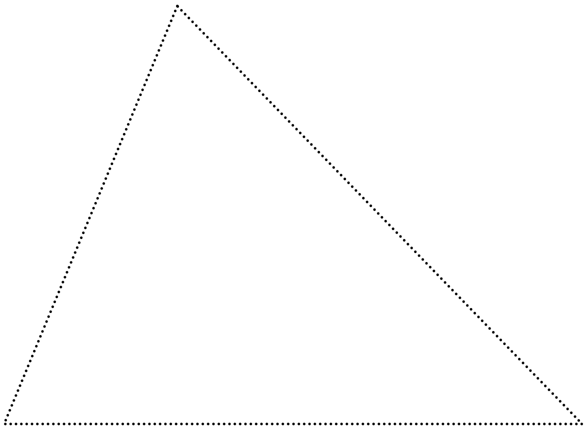


→ →

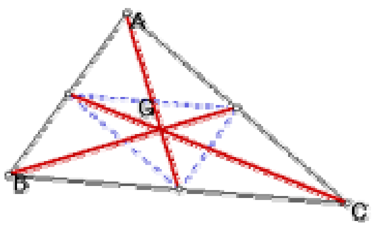
b) Express the area of these three triangles as a percentage of the area of the original triangle.

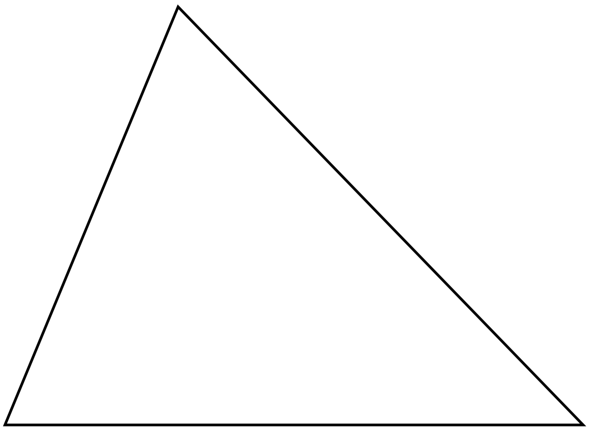
c) What is the fate of the orbit?

d) Draw the **medians** of the original triangle and note how they **intersect** each subsequent triangle on the orbit

Remember

- The medians of a triangle are segments from each vertex to the midpoint of the opposite side.
- The medians always intersect in a single point, called centroid or geocenter.





e) Write down your **conclusion** by completing the sentence below.

The fate of the orbit is **actually**

....

....

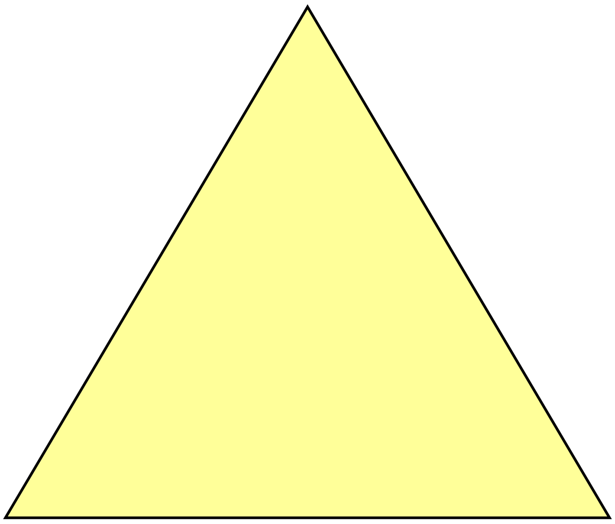
Name: Class: Date:

1-Removals

Start with a filled equilateral triangle
Apply the iteration rule:

- Divide it into four equilateral triangles by marking the midpoints of all three sides and drawing lines to connect the midpoints.
- **Remove** the **interior** of the triangle from the middle.

1. Sketch the first iteration. You will have one **hole** and three smaller **filled** equilateral triangle **remaining**.



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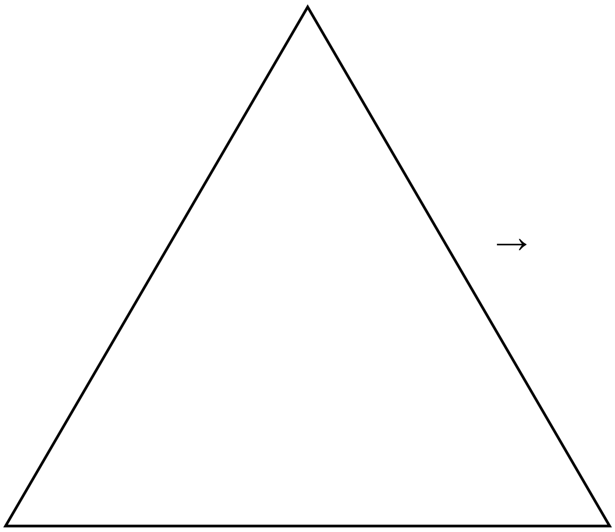
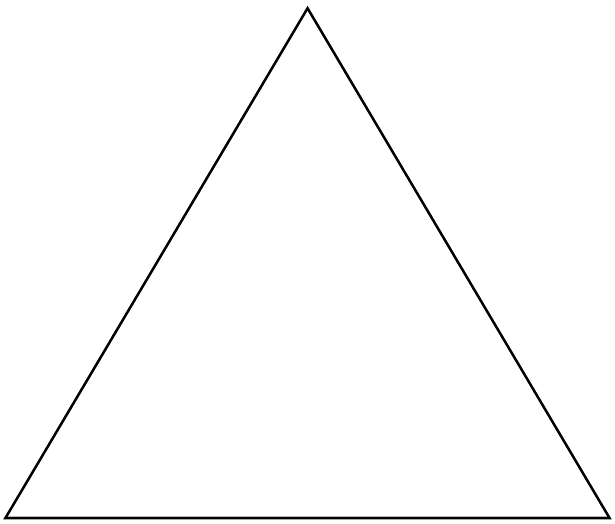


→

First iteration

2. - Sketch the second iteration. You will have to apply three times the iteration rule, once on each of the smaller triangles.

   - Sketch the third iteration.



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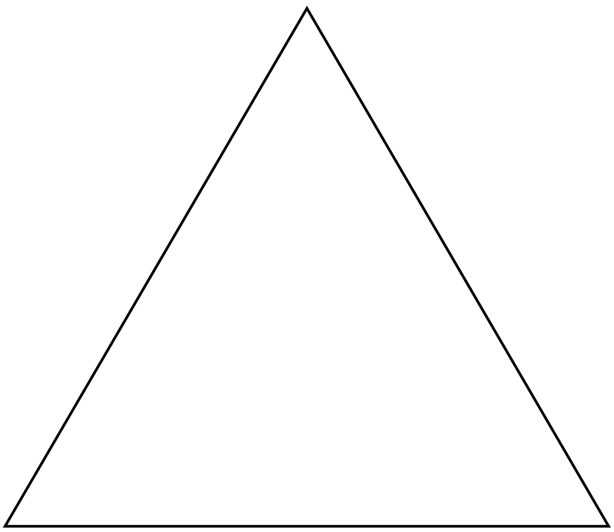


Second iteration

Third iteration

Name: Class: Date:

3. **Look carefully at** your sketches and complete the grid below:

	Number of holes **added**	Total number of holes	Total number of triangles **remaining**
Original square	**0**	**0**	**1**
First iteration	**1**		
Second iteration			
Third iteration			

4. Predict **with your partner** the next row of the grid:

Fourth iteration			

5. What fraction of the triangle is the total number of triangles remaining in the first iteration?

6. What fraction of the triangle is the total number of triangles remaining in the second iteration? **Pay attention** that holes have different sizes.

7. What fraction of the triangle is the total number of triangles remaining in the third iteration?

8. Do you see a **pattern** here? Use this pattern to predict what fraction of the triangle **would be** the total number of triangles remaining in the fourth iteration.

9. Write the fractions in the above questions in order form **greatest** to **least**.

Name: Class: Date:

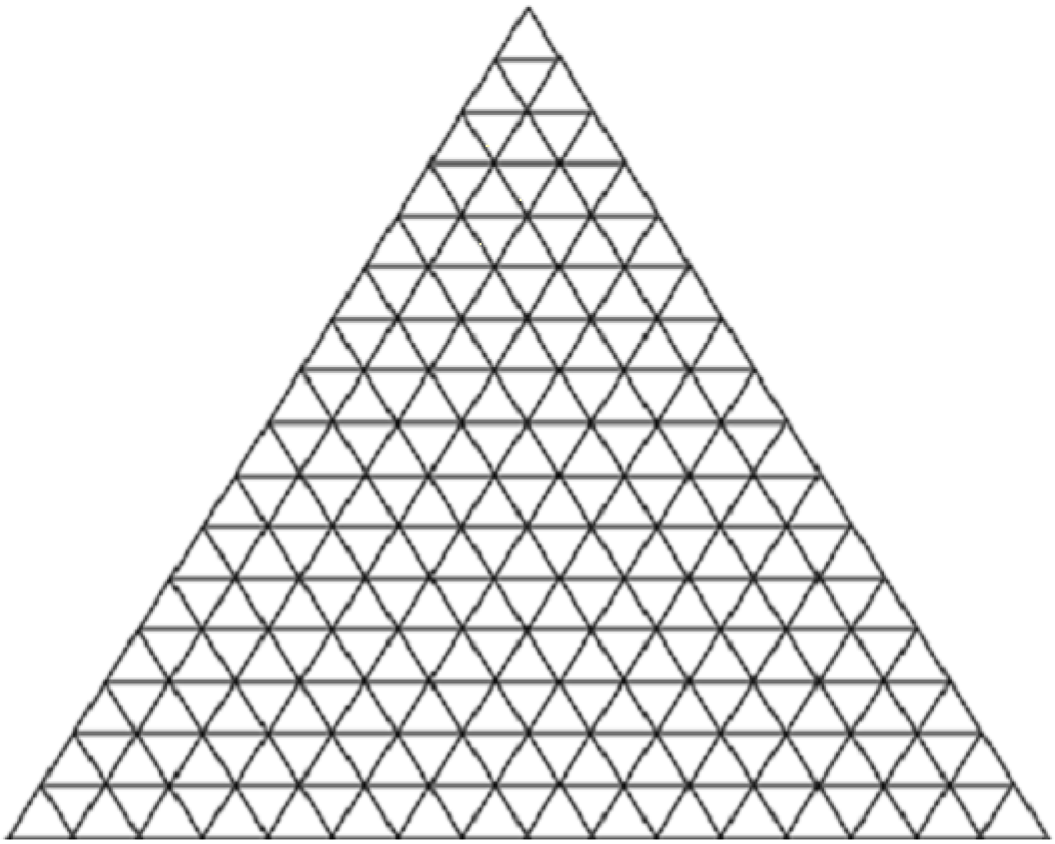
10. Assume the **side length** of the original triangle is **1 unit**.

- Look carefully at your sketches and complete the grid below:

	Original triangle	First iteration	Second iteration	Third iteration
Side length of each triangle remaining (as a fraction)	**1**			

- What **fraction** would be the side length of each triangle remaining on the fourth iteration?

11. Use coloured pencils to sketch below how the fourth iteration looks like.



12. **Confirm** predictions you made before about

- total number of holes and total number of triangles remaining.(4)
- side length of each triangle remaining. (10)

Name: Class: Date:

2-Sierpiński

1. Go to: http://math.rice.edu/~lanius/fractals/sierjava.html

a) Play the game and move from one iteration to the next.
The number of holes at each iteration is shown in the corner.
Stop at the 8th iteration (the number of triangles increases rapidly and a big amount of memory is needed).

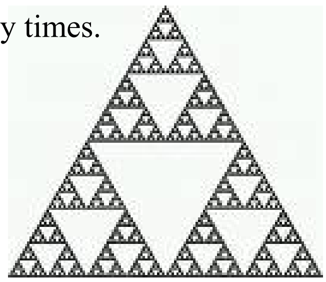
b) Fill in the table below:

Iteration	**1**	**2**						
Total number of holes	**1**	**4**						

c) Do you see a **pattern** here? (Remember **exercise 1. (3) (4)** above) Use this pattern to **predict** the total number of holes in the 9th and in the 10th iteration.

d) In your imagination, you could repeat this rule infinitely many times.
The fate of this orbit is a shape known as

the Sierpiński triangle fractal.



2. Search for information on the Web and complete the following passage by filling in the blanks

.... Sierpiński (.............. - ....) was born in ...., the capital of ....

He described his triangle in the year........................

In 1920 he founded the mathematical journal ...., still edited nowadays by the Polish Academy of Science.

3-Area of the Sierpiński triangle

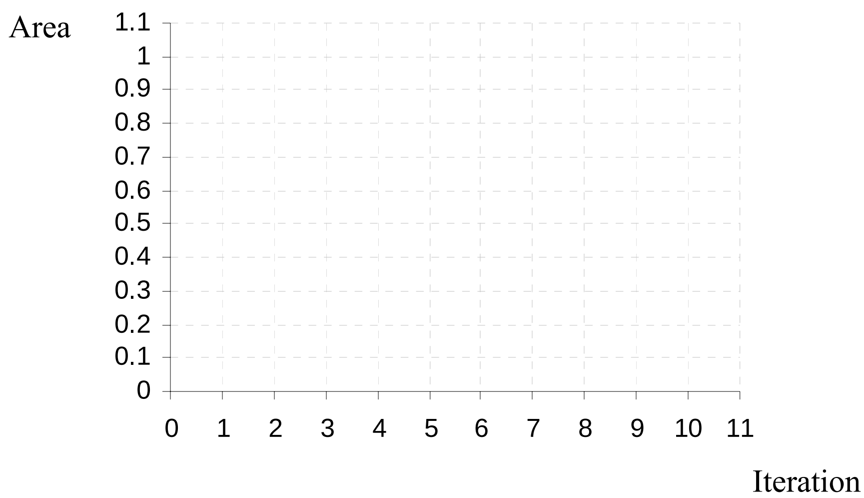
Assume the **area** of the original triangle is **1 square unit.**

a) Look again at exercise 1 above and complete the grid below (round to two decimal places):

	Original triangle	First iteration	Second iteration	Third teration	Fourth iteration
Area as a fraction (in square units)	**1**	$\frac{3}{4}$	$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$		
Area as a decimal (in square units)	**1**	**0.75**			

b) Use a spreadsheet or calculator to compute the area of the next six iterations.

c) Sketch a plot of these eleven points.



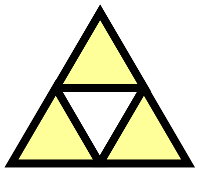
d) What can you explain about the area if we continue iterating?

e) Complete the sentence below:

The area of the Sierpiński triangle **is**..................

Name: Class: Date:

4-Perimeter of the Sierpiński triangle



- Remember the iteration rule:

"Remove the **interior** of the middle triangle".

So the **boundary** of the middle triangle **makes up part** of the **perimeter.**

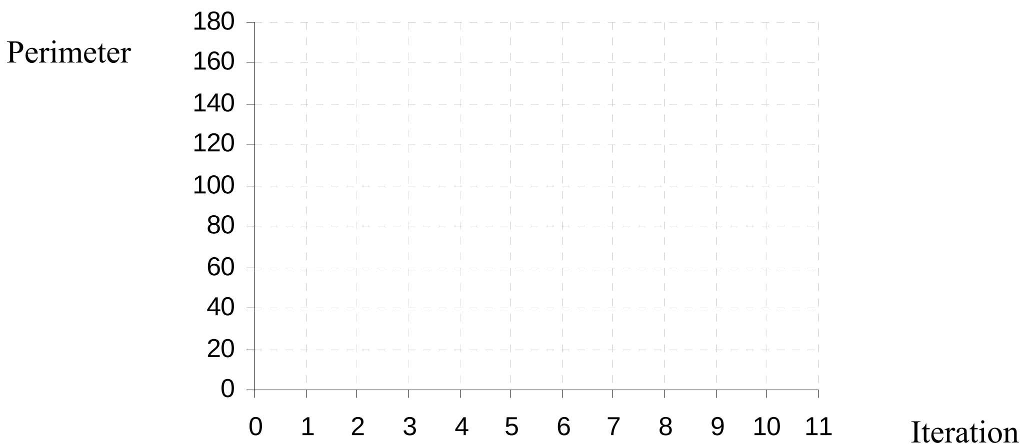
- Assume the **side length** of the original triangle is **1 unit**.

a) Look again at exercise 1 above and complete the grid below:

	Original triangle	First iteration	Second iteration	Third iteration
Side length of each triangle remaining	**1**	$\frac{1}{2}$	$\frac{1}{4}$	
Perimeter of each triangle remaining	$\mathbf{3 \cdot 1 = 3}$	$\mathbf{3} \cdot \frac{1}{2} = \frac{3}{2}$		
Total number of triangles **remaining**	**1**	**3**	**9**	
Perimeter	$\mathbf{3 \cdot 1 = 3}$			

b) Do you see a **pattern** here? Use a spreadsheet or calculator to compute the perimeter of the next seven iterations.

c) Sketch a plot of these eleven points.



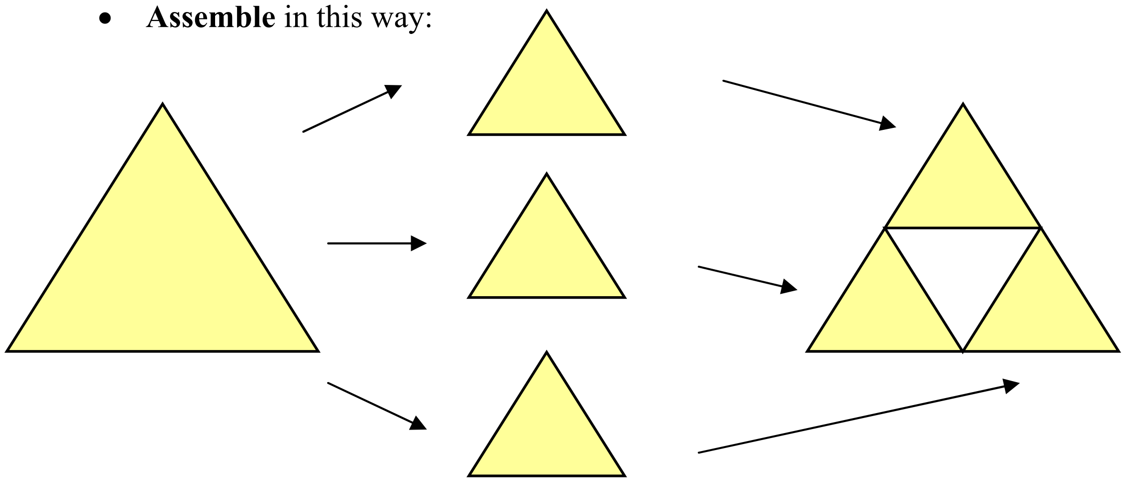
d) What's happening to the perimeter of the Sierpiński triangle?

Name: Class: Date:

5-Copies of copies

Use any computer **drawing program** in this exercise.
Start **again** with a filled equilateral triangle. **Everyone** in class should start with the **same large** triangle.
The iteration rule is:

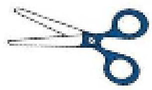
- Make three **copies** of the triangle.
- **Reduce** them by 50%.
- **Assemble** in this way:



Now draw the second, the third and the fourth iteration.
What shape will you get by continuing this process?

....

6-Poster presentation



- **Print** the fourth iteration and **cut out** the triangle.

- **Cooperate** with your classmates by making a **poster** of the **largest** Sierpiński triangle you can create:

  - **Assemble** all of the triangles by **matching** their vertices.

- Ask the following questions:

  - How many pupils are there in your class?
  - How many triangles did you use by making the poster?
  - How many copies do you need to be sure you get a Sierpiński triangle? List the possibilities and explain.

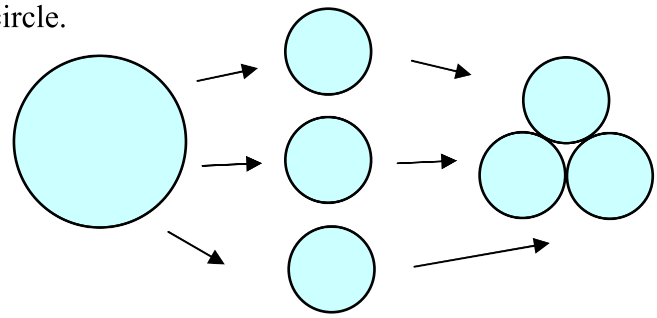
  - If each original triangle has sides of length 15 cm, how many copies would you need to produce a Sierpiński triangle whose **base** is 60 cm long?

Name: Class: Date:

7-Triplicating a circle

Start with a filled circle and follow the rule:

- Make three **copies** of the circle.
- **Reduce** them by 50%.
- **Assemble** in this way:



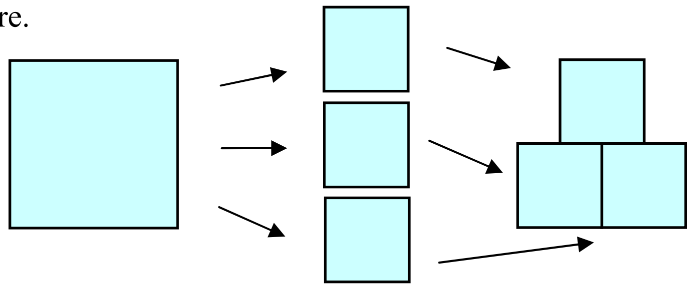
a) Sketch the next three iterations of this rule above.
b) What figure will result if you continue to iterate this rule?

....

8-Triplicating a square

Start with a filled square and follow the rule:

- Make three **copies** of the square.
- **Reduce** them by 50%.
- **Assemble** in this way:



a) Sketch the next three iterations of this rule above.
b) What figure will result if you continue to iterate this rule?

....

9-Further exploration

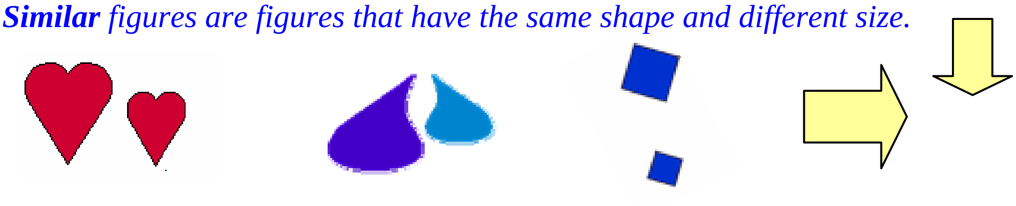
Pick your own photo as your seed or any image you like and proceed with the rule as above.
Organise your results as a poster and **describe** your work using the past "**I have** ...". **Include** your prediction of what you think will result if you continue to iterate this rule indefinitely.

Name: Class: Date:

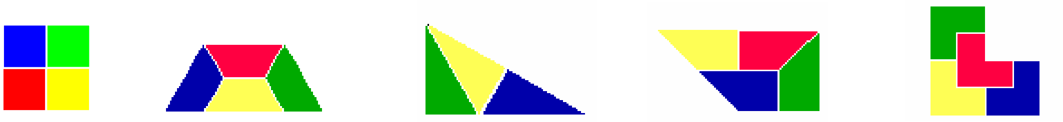
10-Self-similarity

Remember

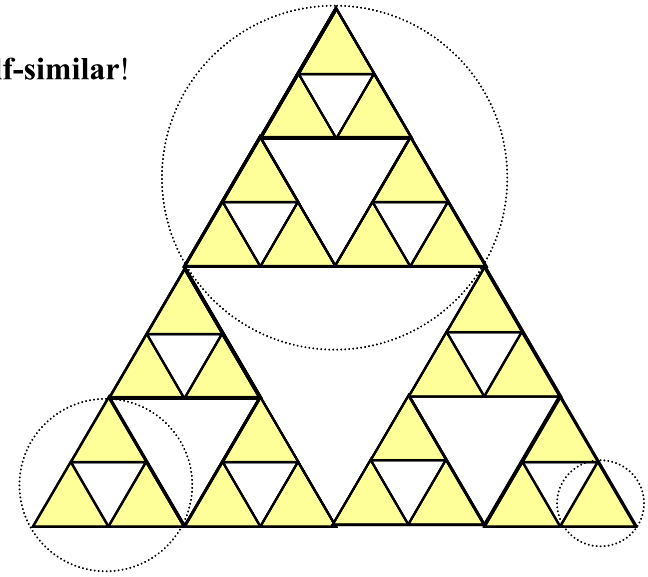
- ***Similar** figures are figures that have the same shape and different size.*



- *A **self-similar** figure is a figure made by small copies of itself.*



The Sierpiński triangle is **self-similar**!



- Take a **magnifying glass** to look more closely at the Sierpiński triangle:

a) How many copies do you see where **magnified by a factor of 2** yield the entire figure?
b) And by a factor of 4?
c) And by a factor of 8?
d) Can you find a pattern here?

- Go to: http://math.rice.edu/~lanius/fractals/selfsim.html and **zoom in**.
The triangle will look the same!

Name: Class: Date:

11-Pascal triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

....

....

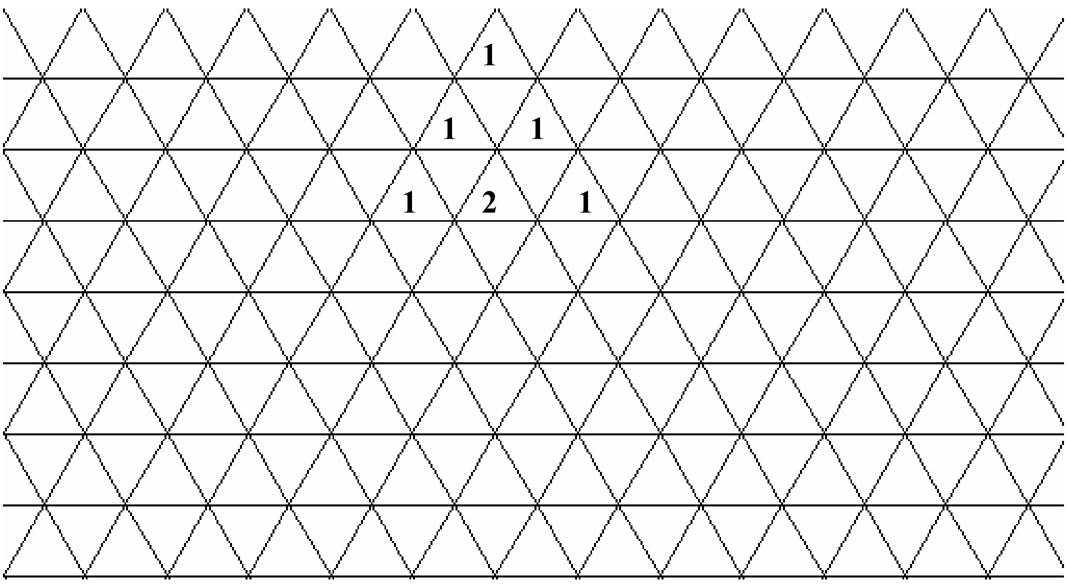
....

a) Do you see the pattern of how the numbers are placed in this triangle?
Fill in the next three rows.

b) This number triangle was known in China and Persia in the eleventh century. Later on the French mathematician Blaise Pascal, among others, studied its properties. In which **century** did Pascal live?

c) Copy the Pascal triangle in the grid below.

d) **Shade** all the little **odd numbered** triangles in the Pascal triangle.



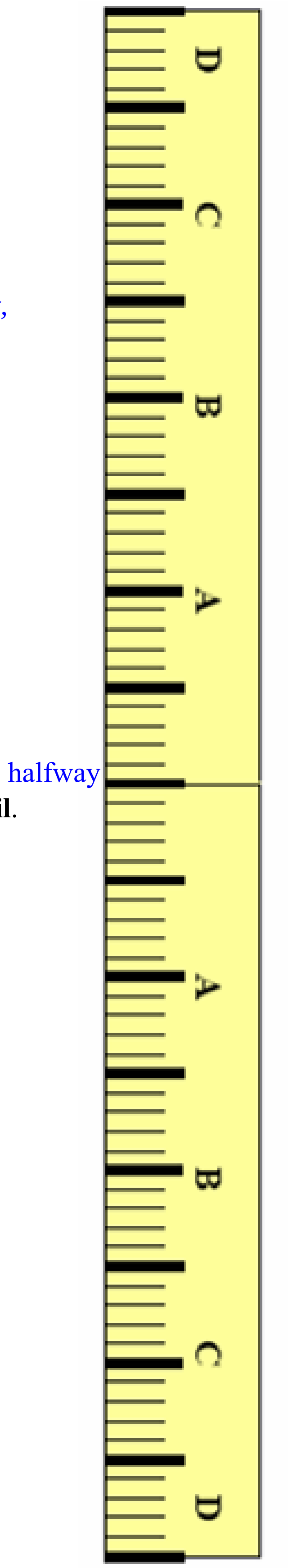
e) What figure did you get?

Name: Class: Date:

12-The chaos game

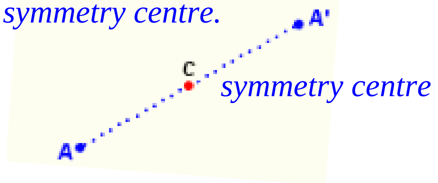

1. Number of players: in pairs.

2. Materials: worksheet, die, pencil and rubber, pen, ruler (better a half-ruler as showed on your right), transparency and marker pen.



Remember

Central symmetry is the reflection of an object through a point, called symmetry centre.



3. Half-ruler: It is handy to use a half-ruler for drawing. You can make your own half-ruler from cardboard.

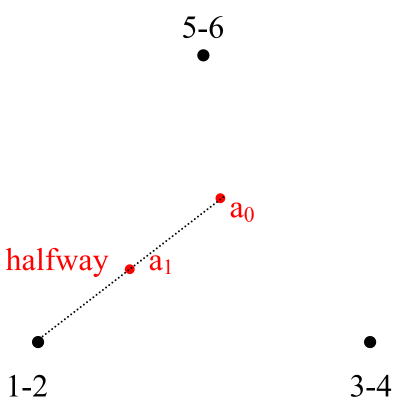
4. How to start playing?

- Place three points at the vertices of an equilateral triangle. Call them 1-2, 3-4 and 5-6.
- Choose **any** point in the interior of the triangle. Use a **pencil**. Call this point a_0.

5. How to play?

- First **roll the die** to determine a **vertex**.
- Then, using a pencil and a half-ruler, move to a new point which is **exactly halfway** between a_0 and the target vertex. Call this point a_1.

6. Example: if you roll a 2

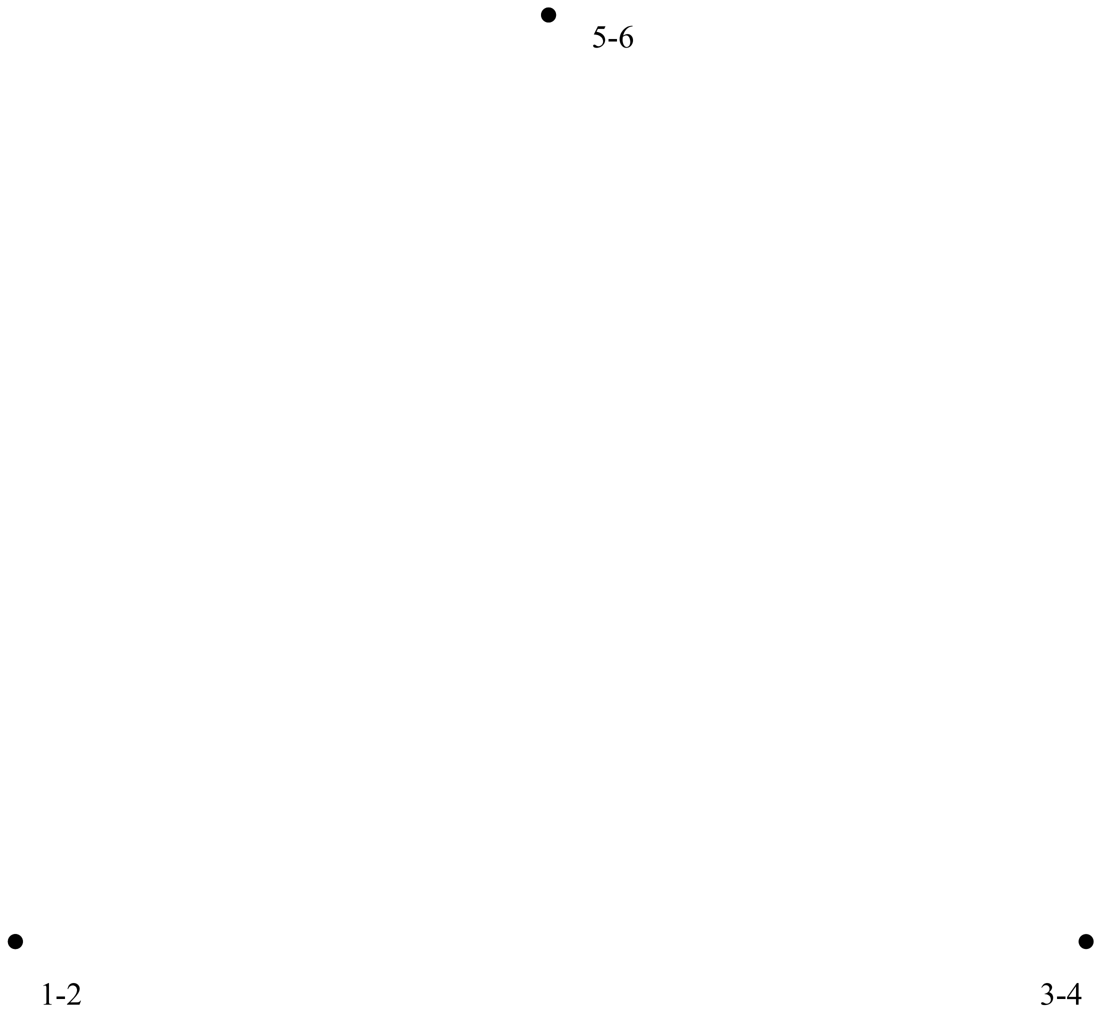


Name: Class: Date:

7. Playing the game

Play the chaos game with the given vertices below.

- Choose a point a_0 somewhere **inside** the triangle.
- Roll the die and plot the point a_1 using your half-ruler and **pencil**.
- Plot the points a_2, a_3, a_4, a_5 and a_6 determined by your die rolls.



- **Switch to a pen** and plot a_7.
- **Erase** the first seven points a_0, a_1, a_2, a_3, a_4, a_5 and a_6.
- Plot the next ten points or more using a pen.
- Compare your results with those of your classmates.
- Copy your points onto a transparency. The rest of the class does the same.

Lining all the transparencies up, do you see a pattern emerging? ....

8. The game on the Web

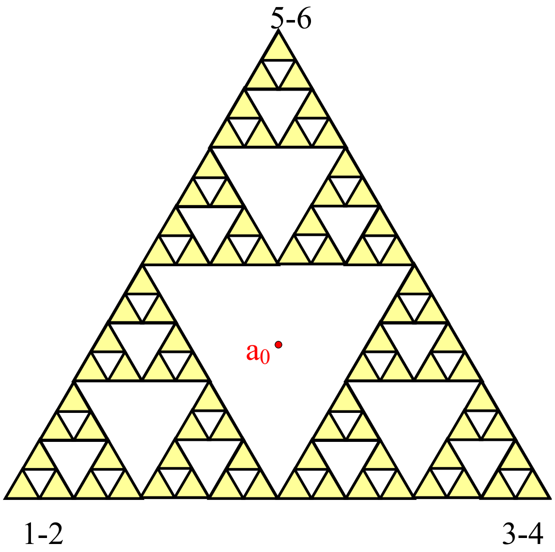
Go to **http://www.jgiesen.de/ChaosSpiel/Spiel1English.html** and play the game plotting one point at a time.
Go to **http://www.jgiesen.de/ChaosSpiel/Spiel1000English.html** and play the game plotting one thousand points at a time.

Name: Class: Date:

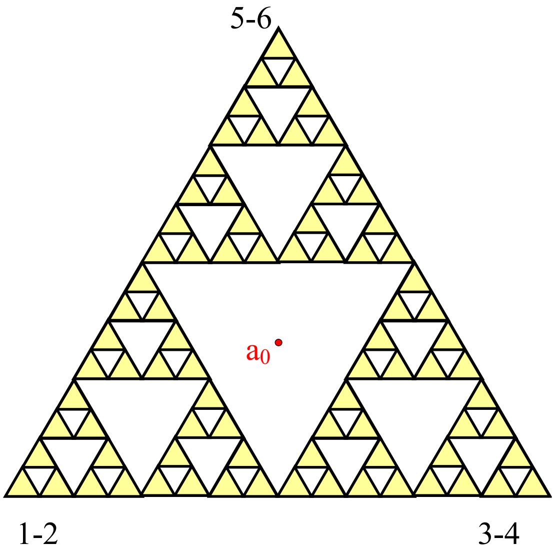
9. Playing the game with the Sierpiński triangle showing

- **Choose** the point a_0 in the middle of the triangle and **plot** the points a_1, a_2 and a_3 in each picture below according to the three rolls of the die indicated.

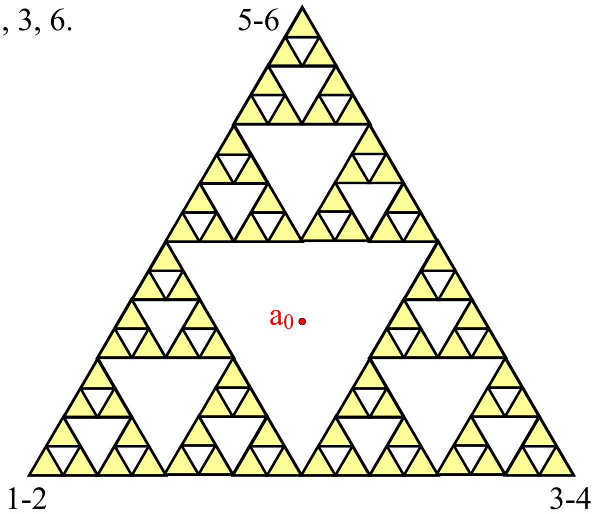
a) You roll 1, 4, 5.



b) You roll 6, 5, 2.



c) You roll 5, 3, 6.



- **Explain** how the points are distributed in the triangles.

Name: Class: Date:

10. Conclusion

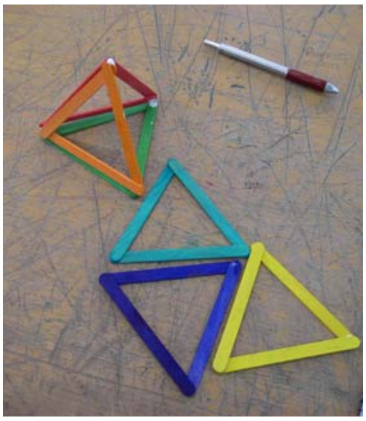
Argue with your partners and your teacher why after playing the chaos game thousands of times appears the image of the Sierpiński triangle.

13-The Sierpiński Tetrahedron

Go to

http://www.toomates.net/llistes/geometria/figures_geometriques/fractals/tetraedre_sierpinski_bastonets_gelat/tetraedre_sierpinski_bastonets_gelat.htm

Look at the pictures and **cooperate** with all of your classmates by building your own Sierpiński Tetrahedron



Materials: ice-cream sticks and glue.

- **At stage 1** you put four small tetrahedra together.

How many small tetrahedra will you need at stage 2?

How many small tetrahedra will you need at stage 3?

How many small tetrahedra will you need at stage 4?

Name: Class: Date:

Can you find a pattern here?

How many small tetrahedra would you need at stage 10?

- Suppose that the length of each stick is 10 cm.

How long will each edge be at stage 1?

How long will each edge be at stage 2?

How long will each edge be at stage 3?

How long will each edge be at stage 4?

Can you find a pattern here?

How long will each edge be at stage 10?

The Sierpiński Tetrahedron is **self-similar**!

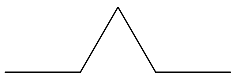
Name: Class: Date:

1-Removals

Here is a geometric iteration:

Seed: a straight line segment
Iteration rule:

- Divide the segment into three equal parts
- **Remove** the middle third
- Replace it with two segments the same length as the section you removed.



1. Start with the straight line segment below and sketch the first and the second iteration.

2. - How many segments does the first iteration have? ....

   - How many segments does the second iteration have? ....

   - How many segments **will** the third iteration have? ....

   - Do you see a **pattern** here? Explain.

Name: Class: Date:

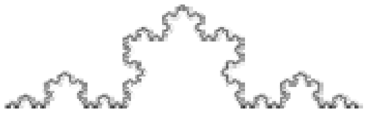
2-Koch curve

1. Go to: http://www.arcytech.org/java/fractals/koch.shtml

a) Play the game and move from one iteration to the next.
Stop at the 11th iteration (it takes a long time to draw and a good memory is needed).

b) How many segments does the eleventh iteration have?

c) In your imagination, you could repeat this rule infinitely many times.
The orbit of this iteration rule tends to famous fractal known as

the Koch curve 

2. Search for information on the Web and complete the following passage by filling in the blanks.

.... Koch (.............. - ....) was born in

...., the capital of ....

He described this curve in the year........................

3. Assume the **length** of the original straight line segment is **1 unit**.

-What **fraction** is the length of each segment at the first iteration?

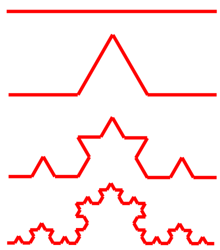
-What fraction is the length of each segment at the second iteration?

-What fraction **will be** the length of each segment at the third iteration?

- Do you see a **pattern** here? Use this pattern to predict what fraction **would be** the length of each segment at the fourth iteration.

Name: Class: Date:

3-Length of the Koch curve



Assume the **length** of the original straight line segment is **1 unit**.

1. Complete the grid below (round to two decimal places):

	Number of segments	Length of each segment	Total length (as a fraction)	Total length (as a decimal)
Original segment	**1**	**1**	**1**	**1**
First iteration	**4**	$\frac{1}{3}$		
Second iteration				
Third iteration				
Fourth iteration				

2. Complete the sentences below:

- The total length of each iteration is .... times the total length of the iteration before.

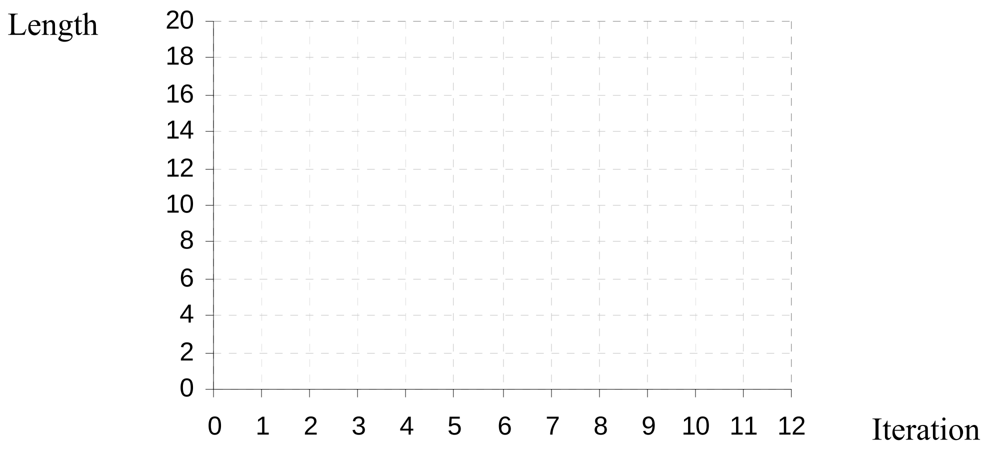
- The total length is **increasing** by a scale factor of.....................

- The total length of the fifth iteration is...units.

3. Use a spreadsheet or calculator to compute the total length of the next seven iterations.

Name: Class: Date:

4. Sketch a plot of these twelve points.

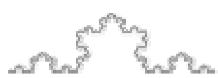


5. How many iterations would it take to obtain a total length of 100 units, or as **close to** 100 as you can get?

6. How many iterations would you have to draw to guarantee that the length is **larger** than 200 units?

7. Compare with your partner the answers above to number 5 and number 6. What can you explain about it?

8. Write down a formula to calculate the length of the curve at the thousandth iteration.



9. What can you explain about the length if we continue iterating?

Name: Class: Date:

4-Copies of copies

Use any computer **drawing program** in this exercise.
Start **again** with a straight line segment. **Everyone** in class should start with the **same size** segment.
The iteration rule is:

- Make four **copies** of the segment, each **reduced** to one-third the original size.

- **Assemble** in this way: 

1. From left to right the second segment is **rotated counter clockwise** and the third one is rotated **clockwise.**
   What is the angle of rotation? ....

2. Now draw the second, the third and the fourth iteration.

3. Go to: http://www.xtec.cat/~dobrador/abeam/koch.swf

   What shape will you get by continuing this process?...

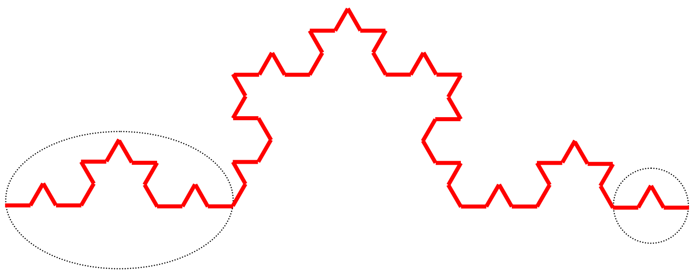
5-Frieze presentation

1. **Print** the fourth iteration.

2. **Cooperate** with your classmates by making an ornamental **frieze** to decorate your classroom.

6-Self-similarity

The Koch curve is **self-similar**!



- Take a **magnifying glass** to look more closely at the Koch curve:

a) How many copies do you see where **magnified by a factor of 3** yield the entire figure?
b) And by a factor of 9?

- Go to**:** http://www.jimloy.com/fractals/koch.htm and look at the movie **Zooming in on the Koch Curve**.
The curve looks the same!

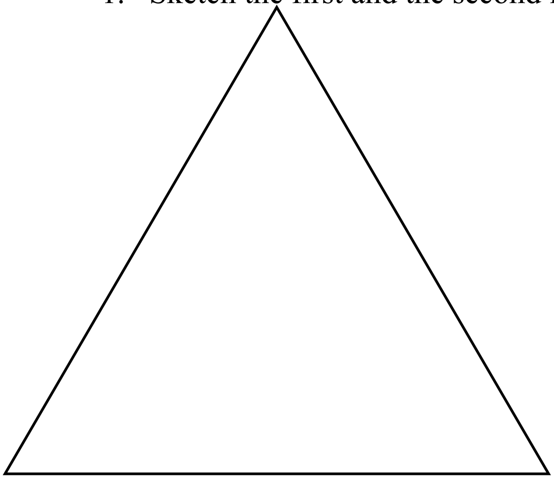
Name: Class: Date:

7-The Koch Snowflake

Start with an equilateral triangle.
Apply the iteration rule:

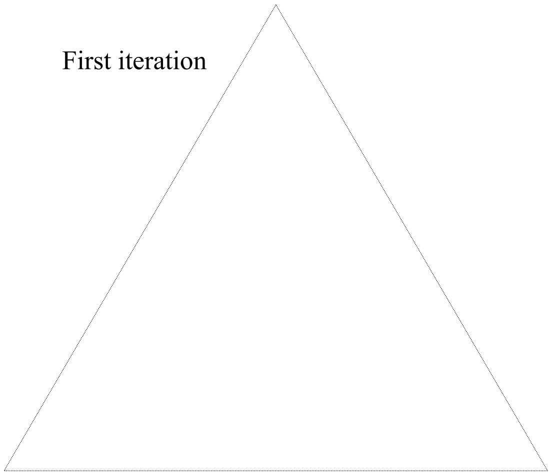
- Divide **each side** of the triangle into three equal parts.
- **Remove** the middle third.
- Replace it with two segments the same length as the section you removed.

1. Sketch the first and the second iteration.

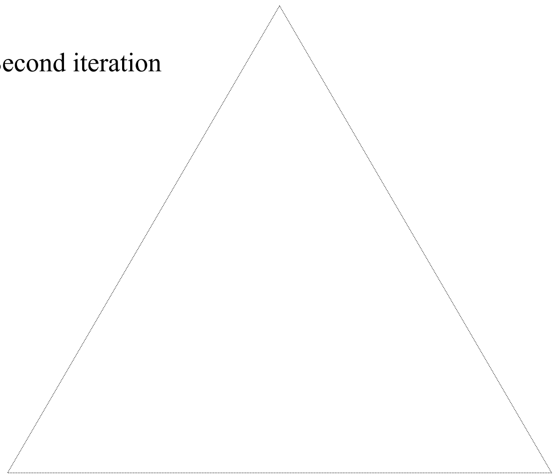


→

First iteration



Second iteration



2. - How many sides does the first **polygon** have? ....
   - How many sides does the second polygon have? ....
   - How many sides **will** the third iteration have? ....
   - Do you see a **pattern** here? Explain.

Name: Class: Date:

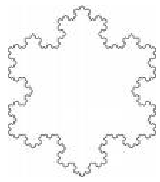
3. Go to: http://math.rice.edu/~lanius/frac/koch/koch.html

a) Play the game and move from one iteration to the next.

b) How many sides does the sixth iteration have?

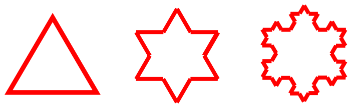
c) In your imagination, you could repeat this rule infinitely many times.
The orbit of this iteration rule tends to famous fractal known as

the Koch Snowflake curve



8-Perimeter of the Koch Snowflake

Assume the **perimeter** of the original triangle is **3 units**.



1. Complete the grid below (round to two decimal places):

	Number of sides	Length of each side	Perimeter (as a fraction)	Perimeter (as a decimal)
Original triangle	**3**	**1**	$\mathbf{3 \cdot 1 = 3}$	**3**
First iteration	$\mathbf{4 \cdot 3 = 12}$	$\frac{1}{3}$		
Second iteration				
Third iteration				
Fourth iteration				

2. Complete the sentences below:

- The perimeter of each figure is .... times the perimeter of the figure before.

- The perimeter is **increasing** by a scale factor of....................

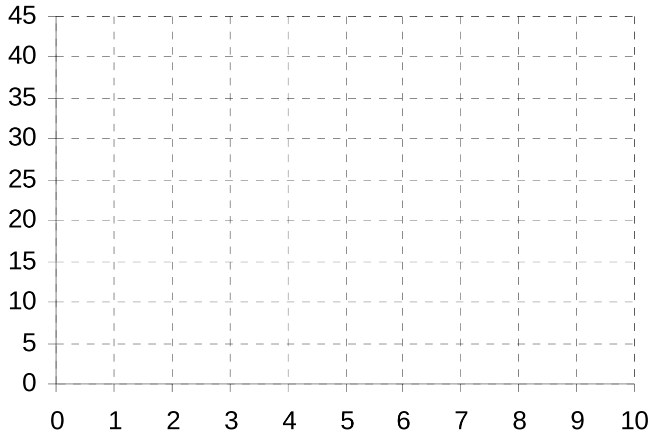
- The perimeter of the fifth iteration is..units.

Name: Class: Date:

3. Use a spreadsheet or calculator to compute the perimeter of the next five iterations.

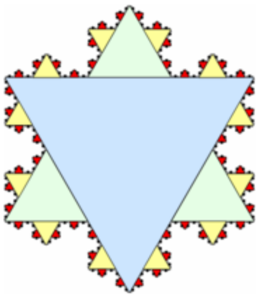
4. Sketch a plot of these ten points.

Perimeter



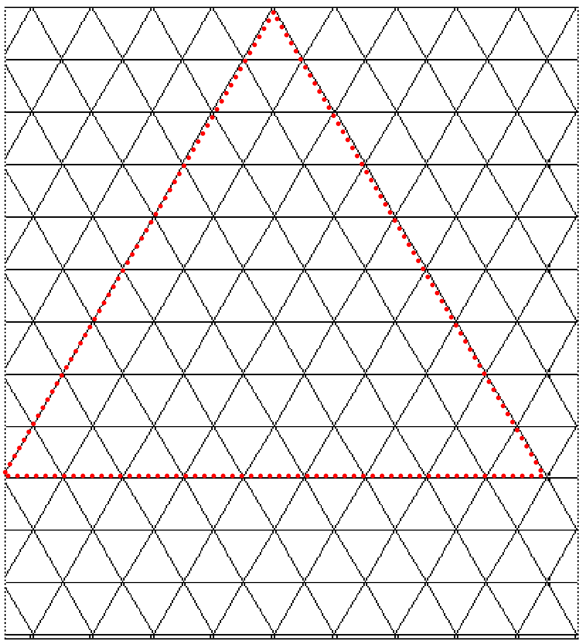
Iteration

5. What can you explain about the perimeter if we continue iterating?



8-Area of the Koch Snowflake

1. Use coloured pencils to sketch below how the second iteration looks like.



Name: Class: Date:

2. Let's measure the area using the small triangles of our grid as units.

1 area unit = 

	Number of triangles added	Area of each triangle added	Amount of area added	Total area
Original triangle				
First iteration				
Second iteration				

3. Complete the sentences below:

- The number of triangles added is .... times the previous one.

- The area of each triangle added is .... the previous one.

- The amount of area added is **increasing** by a scale factor of.....................

4. Predict **with your partner** the next five rows of the table:

5. What is remarkable about the area and the perimeter of this shape?

6. Go to: http://math.rice.edu/~lanius/frac/koch3.html and check your answers.

Name: Class: Date:

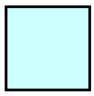
1-Dimension

Complete the sentences with the words in the box.

one	two	three	length	width	depth

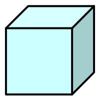
1. A straight line segment has ……… dimension: …………………..

2. The interior of a square has ……… dimensions: ………………….. and …………



3. The interior of a cube has ………... dimensions: ………………….., ……………

and ……………..

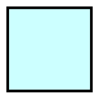


2-The Sierpiński triangle

1. Take the straight line segment below and double its length.

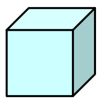
How many copies of the original segment do you get?

2. Take the square and double its length and width



How many copies of the original square do you get?

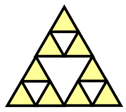
3. Take the cube and double its length, width and height.



How many copies of the original cube do you get?

Name: Class: Date:

4. Take the Sierpiński triangle and double the length of his sides



How many copies of the original triangle do you get?

5. Organise your information into the table

Figure	**Dimension**	**Number of copies**	**Number of copies as a power of two**
straight line segment			
square			
cube			
Sierpiński triangle			

Do you see a pattern here? ……………………………………………………………

…………………………………………………………………………………………

Experiment with the **exponent key** of your calculator to find out the fractal dimension of the Sierpiński triangle (**correct** to two decimal places).

…………………………………………………………………………………………..

3-The Sierpiński Tetrahedron

- Take the Sierpiński tetrahedron and double the edge length

- How many copies of the original tetrahedron do you get?

- Write down the number of copies as a power of two.

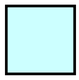
- What is the fractal dimension of the Sierpiński tetrahedron?

Name: Class: Date:

4- The Koch curve

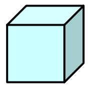
1. Take the straight line segment below and triple its length.

How many copies of the original segment do you get?



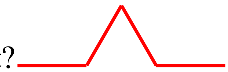
2. Take the square and triple its length and width
How many copies of the original square do you get?

3. Take the cube and triple its length, width and height.
How many copies of the original cube do you get?



4. Take the Koch curve and magnify it by a factor of 3

How many copies of the original curve do you get?



5. Organise your information into the table

Figure	**Dimension**	**Number of copies**	**Number of copies as a power of three**
straight line segment			
square			
cube			
Koch curve			

Do you see a pattern here? ………………………………………………………………

………………………………………………………………………………………………

Experiment with the **exponent key** of your calculator to find out the fractal dimension of the Koch curve (**correct** to two decimal places).

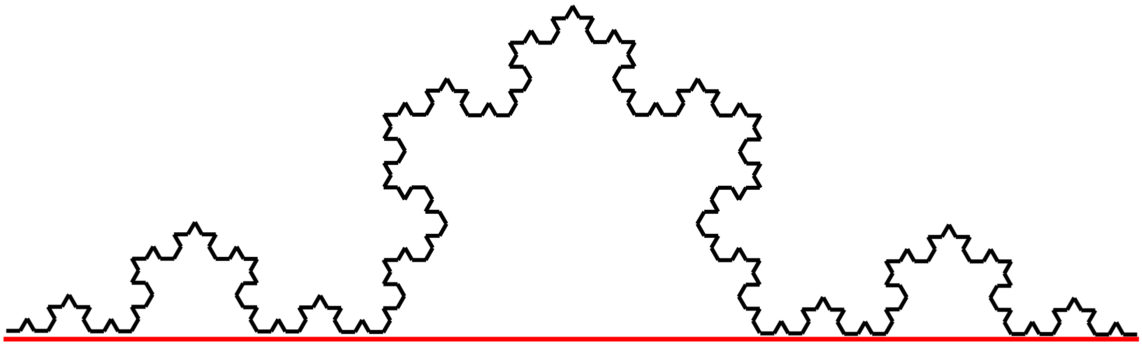
………………………………………………………………………………………………..

6. Compare the fractal dimension of the Koch curve to the fractal dimension of the Sierpiński triangle.

Name: Class: Date:

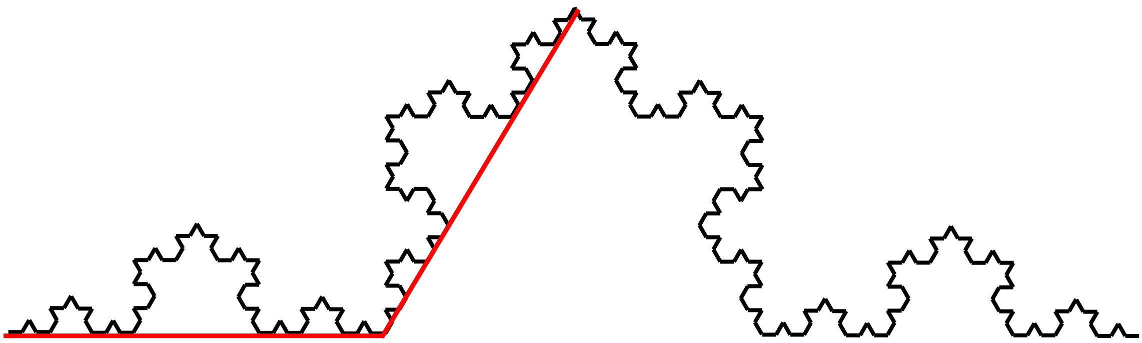
5- Measuring the Koch curve

Materials: Make your own ruler by cutting a strip of paper 15 cm = 1 unit long.
A compass would be useful too.



1 unit

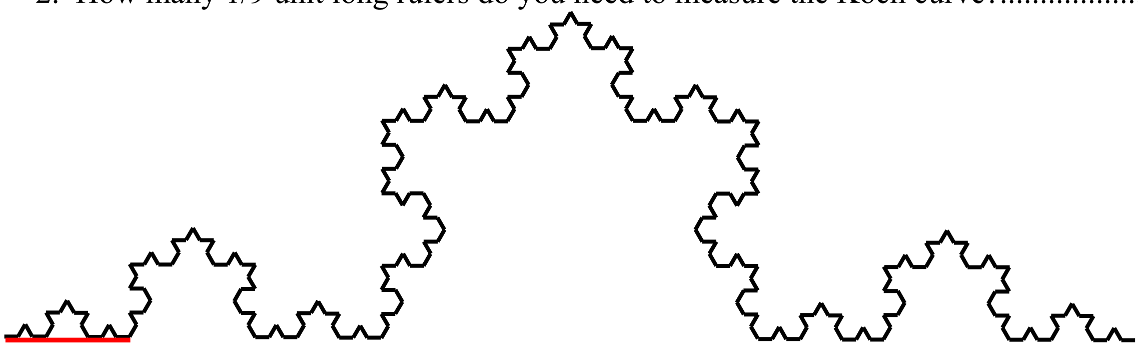
1. Folding your previous ruler into three parts measure the Koch curve.



1/3 unit

How many 1/3 unit long rulers do you need?…………………

2. How many 1/9 unit long rulers do you need to measure the Koch curve?.................



1/9 unit

3. Do you see a pattern here?

Name: Class: Date:

4. Complete the grid below:

	Rulers length	**Num of rulers**
Stage 0	1	1
Stage 1	$\frac{1}{3} = 0.\overset{\frown}{3}$	
Stage 2	$\left(\frac{1}{3}\right)^2 = \frac{1}{9} = 0.\overset{\frown}{1}$	
Stage 3	$\left(\frac{1}{3}\right)^3 = \frac{1}{27} = 0.0370...$	
Stage 4		

The smaller the ruler, the more rulers we need to measure the length of the Koch curve.

5. Experiment with the **exponent key 10** ^{x} of your calculator to **rewrite in base 10** the entries in the table above (**correct** to two decimal places):

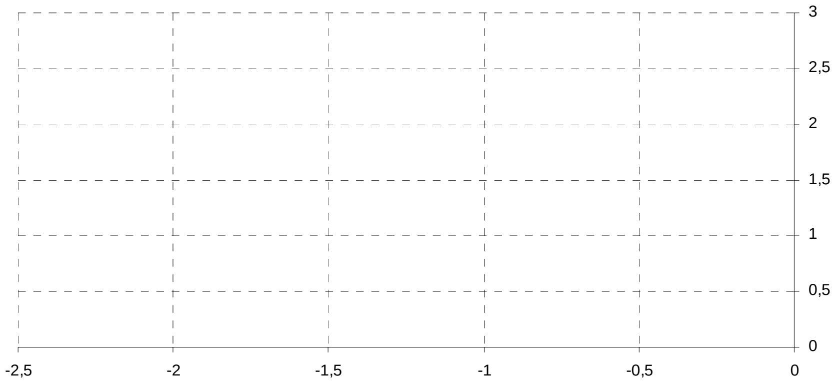
	Rulers length	**Num of rulers**
Stage 0	10^0	10^0
Stage 1	$10^{-0.47}$	
Stage 2		
Stage 3		
Stage 4		

Name: Class: Date:

6. Complete the table:

Exponent of ruler length in base 10	0	-0.47			
Exponent of Num of rulers in base 10	0				

7. Sketch the graph **of the exponents** in the table above:



Exponent of Num. of rulers in base 10

Exponent of ruler length in base 10

8. - Do these points lie on a straight line?

- Calculate the ratio $\dfrac{Exponent\ \ of\ \ Num.\ of\ \ rulers\ \ in\ \ base\ \ 10}{Exponent\ \ of\ \ ruler\ \ lenght\ \ in\ \ base\ \ 10}$

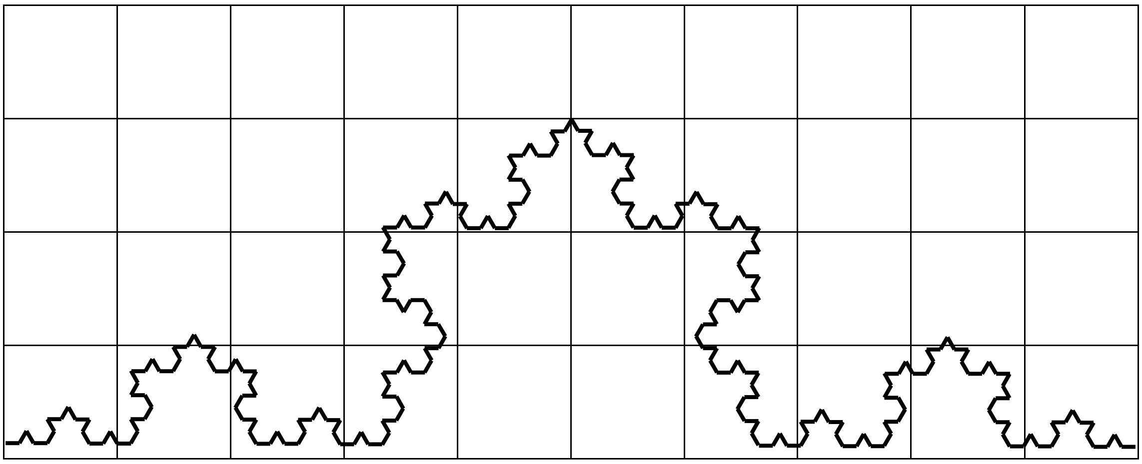
$\dfrac{\quad}{-0{,}47} \approx$ $\qquad$ $\dfrac{\quad}{\quad} \approx$ $\qquad$ $\dfrac{\quad}{\quad} \approx$ $\qquad$ $\dfrac{\quad}{\quad} \approx$

- Do you recognise these values?

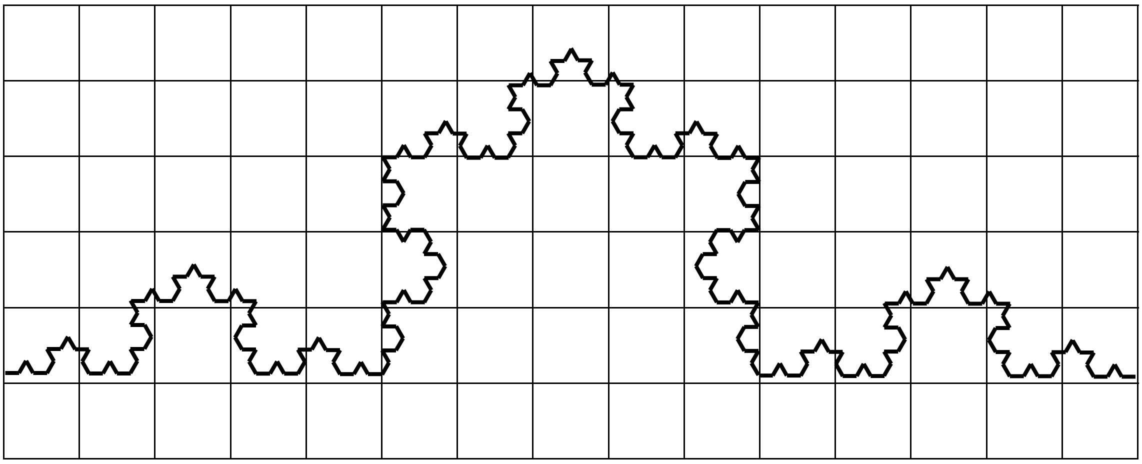
Name: Class: Date:

6- The Box Counting method

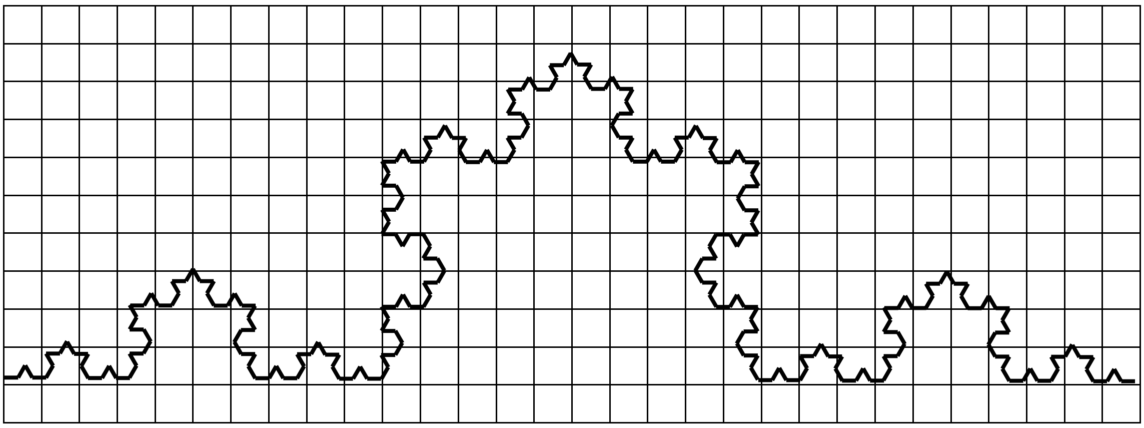
1. Paint the boxes below that contain some of the Koch curve:



10-sized grid



15-sized grid



30-sized grid

Name: Class: Date:

2. Count the number of painted boxes and fill in the table below:

Grid size	10		
Num. of boxes needed to cover the Koch curve			

3. Experiment with the **exponent key 10^x** of your calculator to **rewrite in base 10** the entries in the table above (**correct** to two decimal places):

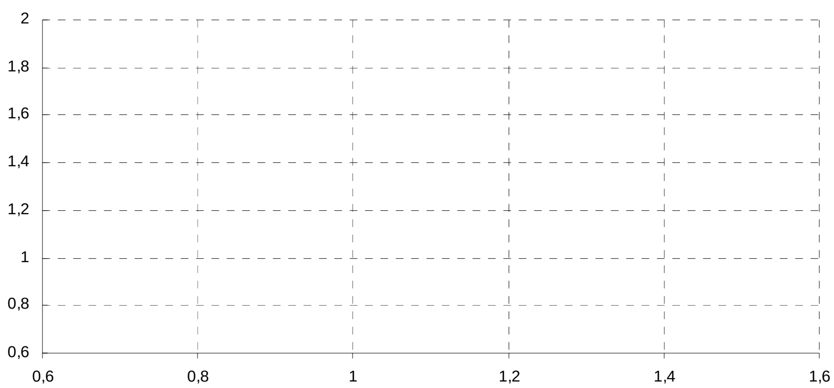
Grid size	10^1		
Num. of boxes needed to cover the Koch curve			

4. Complete the table:

Exponent of Grid size in base 10	1		
Exponent of Num. of boxes in base 10			

5. Sketch the graph **of the exponents** in the table above:

Exponent of Num. of boxes in base 10



Exponent of Grid size in base 10

Name: Class: Date:

6. - Do these points lie on a straight line?

- Calculate the ratio $\dfrac{Exponent\ \ of\ \ Num.\ of\ \ boxes\ \ in\ \ base\ 10}{Exponent\ \ of\ \ Grid\ \ size\ \ in\ \ base\ 10}$

$\dfrac{\qquad}{1} \approx$ $\dfrac{\qquad}{\qquad} \approx$ $\dfrac{\qquad}{\qquad} \approx$

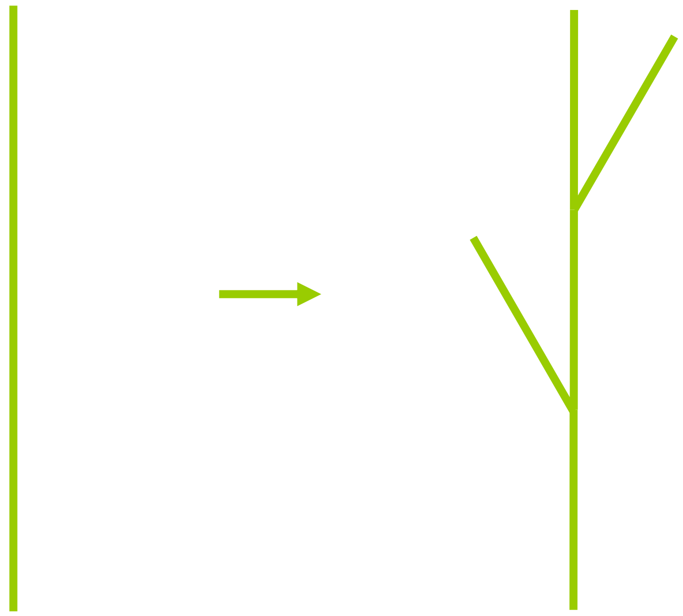
- Do you recognise these values?

Name: Class: Date:

1-Plant-growth

1. Use any computer **drawing program** in this exercise.
Start with a straight line segment. **Everyone** in class should start with the **same long** segment.
Apply the following iteration rule:

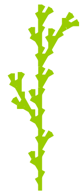
- Reduce the segment by one third and make five copies.
- Rotate one copy 30º clockwise and another 30º anticlockwise.
- Assemble in this way:



Now draw the second, the third and the fourth iteration.

Name: Class: Date:

2. If we continue iterating we get a fractal plant.



- To investigate self-similarity and its fractal dimension, fill in the blanks:

After magnifying by a factor of 3 we get ……….. copies of the original plant.

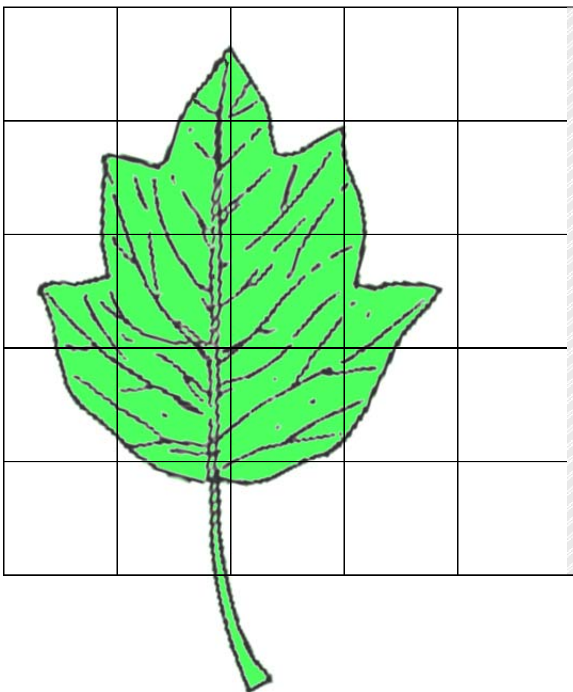
So the fractal dimension will be the exponent *d* such that ……………………..

- Experiment with the **exponent key** of your calculator to find out the fractal dimension (**correct** to two decimal places).

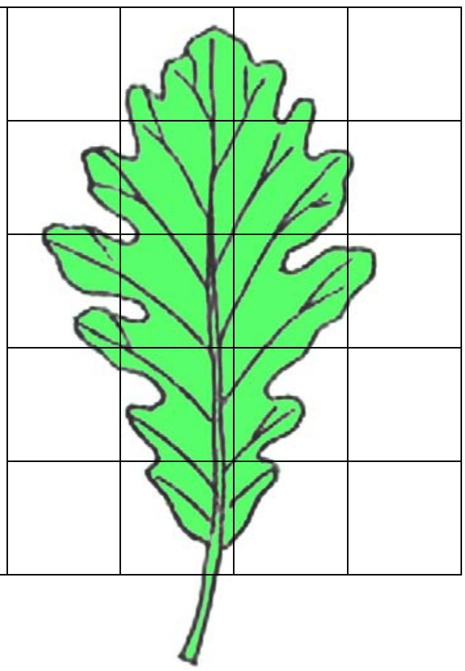
- Compare to the fractal dimension of the Koch curve.

2-Leaf outline

1. Paint the boxes below that contain some of the leaves:

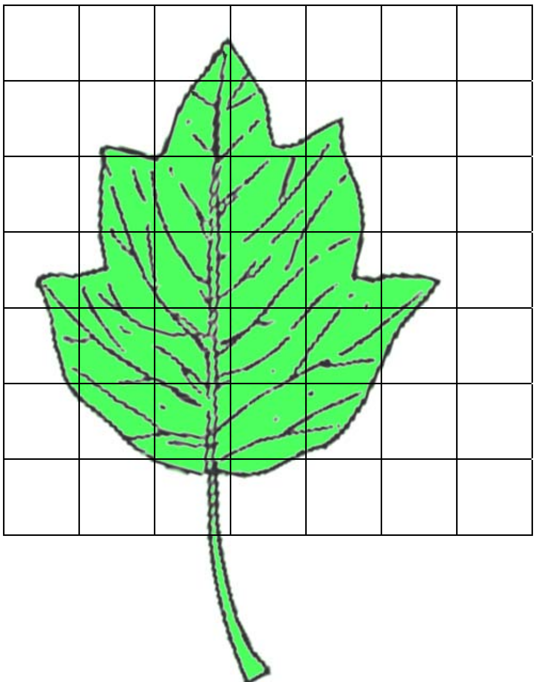


5-sized grid

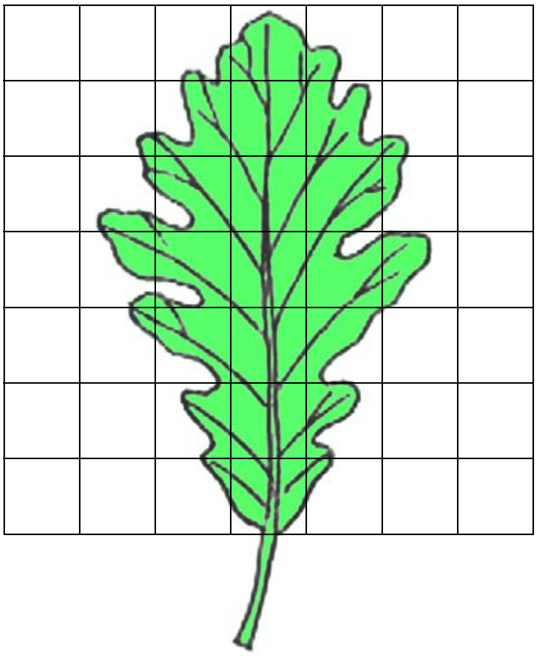


5-sized grid

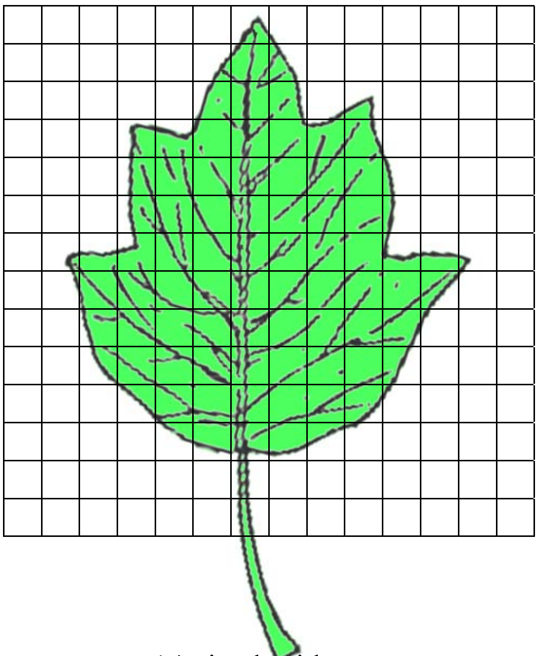
Name: Class: Date:



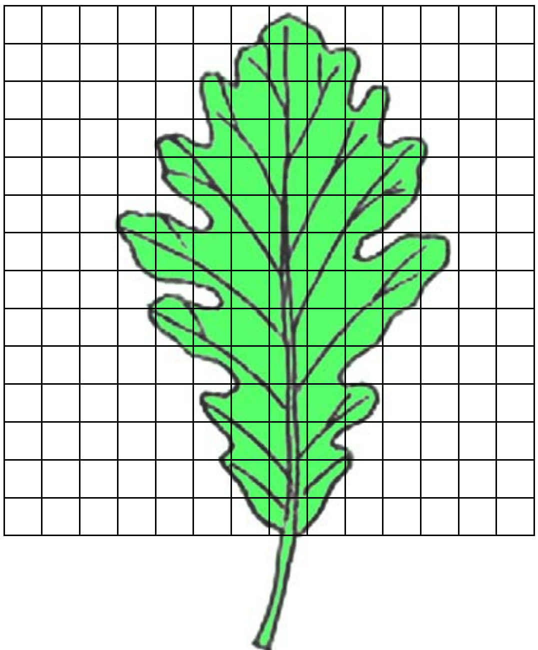
7-sized grid



7-sized grid



14-sized grid



14-sized grid

Name: Class: Date:

2. Count the number of painted boxes and fill in the table below:

Grid size	5		
Num. of boxes covering the leaf contour 1			
Num. of boxes covering the leaf contour 2			

3. Experiment with the **exponent key 10** ^{x} of your calculator to **rewrite in base 10** the entries in the table above (**correct** to two decimal places):

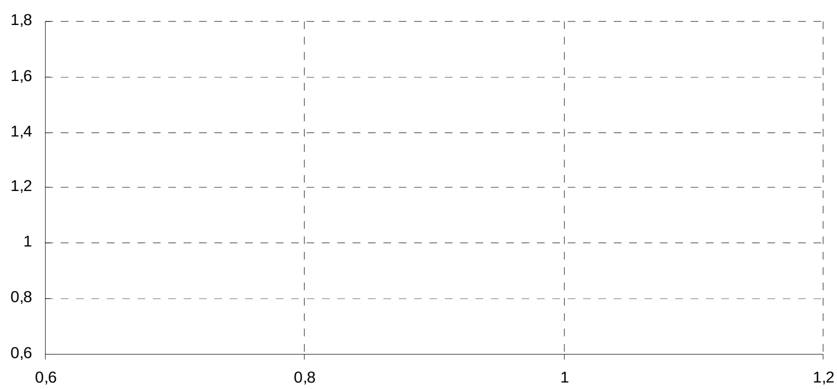
Grid size	$10^{0.69}$		
Num. of boxes covering the leaf contour 1			
Num. of boxes covering the leaf contour 2			

4. Complete the table:

Exponent of Grid size in base 10	0.69		
Leaf 1:Exponent of Num. of boxes in base 10			
Leaf 2:Exponent of Num. of boxes in base 10			

5. Sketch the graphs **of the exponents** in the table above (use two colours):

Exponent of Num. of boxes in base 10



Exponent of Grid size in base 10

Legend: ☐ Leaf1 ☐ Leaf 2

Name: Class: Date:

6. Calculate the ratio $\frac{Exponent\ \ of\ \ Num.\ of\ \ boxes\ \ in\ \ base\ \ 10}{Exponent\ \ of\ \ Grid\ \ size\ \ in\ \ base\ \ 10}$

Leaf 1:

$\frac{\quad}{0.69} \approx$ $\qquad$ $\frac{\quad}{\quad} \approx$ $\qquad$ $\frac{\quad}{\quad} \approx$

Leaf 2:

$\frac{\quad}{0.69} \approx$ $\qquad$ $\frac{\quad}{\quad} \approx$ $\qquad$ $\frac{\quad}{\quad} \approx$

7. What are the fractal dimensions of these leaves outlines?

8. What does the fractal dimension measure? Argue with your partner.

3-Further exploration

Cooperative group: Three or four classmates.

Materials:
- Leaves of different shapes from the park next to your school.
- Different sized square grids printed on transparencies.
- Calculator, cardboard, glue, pen, coloured pencils, markers…

Work: Calculate the fractal dimension of the outline of your leaves.

Poster presentation:
- Order your leaves by fractal dimension from the lowest to the highest.
- Glue them on a cardboard.
- Write down its fractal dimension.
- Present your poster to the rest of the class.

Name: Class: Date:

Name: Class: Date:

Name: Class: Date:

Name: Class: Date:

4-Cauliflower

Broccoli

Cauliflower

- Cauliflower and broccoli are fractals because they branch off into smaller and smaller pieces, which are similar in shape to the original.

Cooperative group: Three or four classmates.

Materials:
- Cauliflower or broccoli.
- Knife and magnifying glass.

Aim: Explore self-similarity and determine the fractal dimension of cauliflower and broccoli.

Procedure:
- Count the number of branches growing off the main trunk.
- Cut off these branches.
- They are copies of the whole. Estimate on average the magnification factor.
- Repeat the previous steps on all of these branches. Take the mean.
- Repeat again. Eventually, you will need a magnifying glass to see the last branches you cut off.

- Complete the sentence:
Every branch carries around ……………..……..branches …………....times smaller.

The fractal dimension will be approximately the exponent *d* such that

$$\boxed{(\textit{Mean of number of branches})^d = \textit{Mean of magnification factors}}$$

- Experiment with the **exponent key** of your calculator to find out the fractal dimension (**correct** to two decimal places).

- Compare your dimension to the dimension from other groups.

Name: Class: Date:

5-Coastline

Cooperative group: Three or four classmates.

Aim: Calculate the fractal dimension of *Estany de Banyoles* (*Catalonia*).

Materials:
- Map of the coastline
- Make your own ruler by cutting a strip of paper 16 cm = 1 unit long.

1 unit

- Compasses would be useful too.

Procedure: The Ruler method.

- Measure the length of the coastline using a collection of rulers whose lengths get shorter and shorter.
- **To minimize error, average the results from your group.**
- Rewrite the data as a power of 10.
- Plot the exponent data "length of ruler versus number of rulers needed".
- You get a set of points that is almost linear.

1. After measuring the coastline from the map below, complete the table:

	Rulers length	**Mean of Num of rulers**
Stage 0	1	1
Stage 1	$\frac{1}{2} = 0.5$	
Stage 2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$	
Stage 3	$\left(\frac{1}{2}\right)^3 = \frac{1}{8} =$	
Stage 4		

Name: Class: Date:

1 unit

1/2 units

1/4 units

1/8 units

1/16 units

1:10.000

Name: Class: Date:

2. Experiment with the **exponent key 10 ^{x}** of your calculator to **rewrite in base 10** the entries in the table above (**correct** to two decimal places):

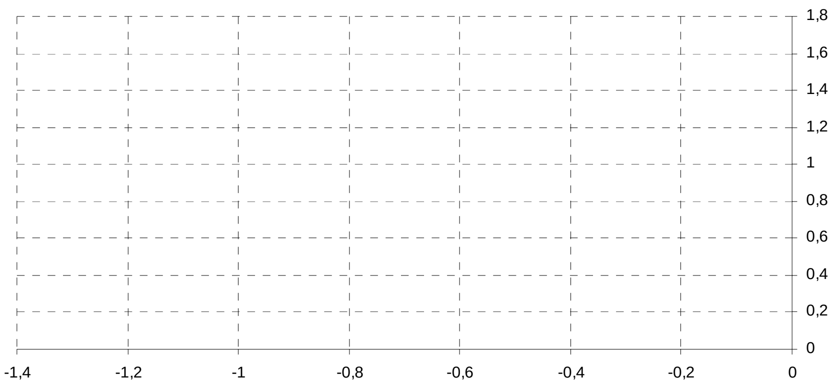
	Rulers length	**Num of rulers**
Stage 0	10^0	10^0
Stage 1	$10^{-0.30}$	
Stage 2		
Stage 3		
Stage 4		

The smaller the ruler, the more rulers we need to measure the coastline.

3. Complete the table:

Exponent of ruler length in base 10	0	-0.30			
Exponent of Num of rulers in base 10	0				

4. Sketch the graph **of the exponents** in the table above:



Exponent of Num. of rulers in base 10

Exponent of ruler length in base 10

Name: Class: Date:

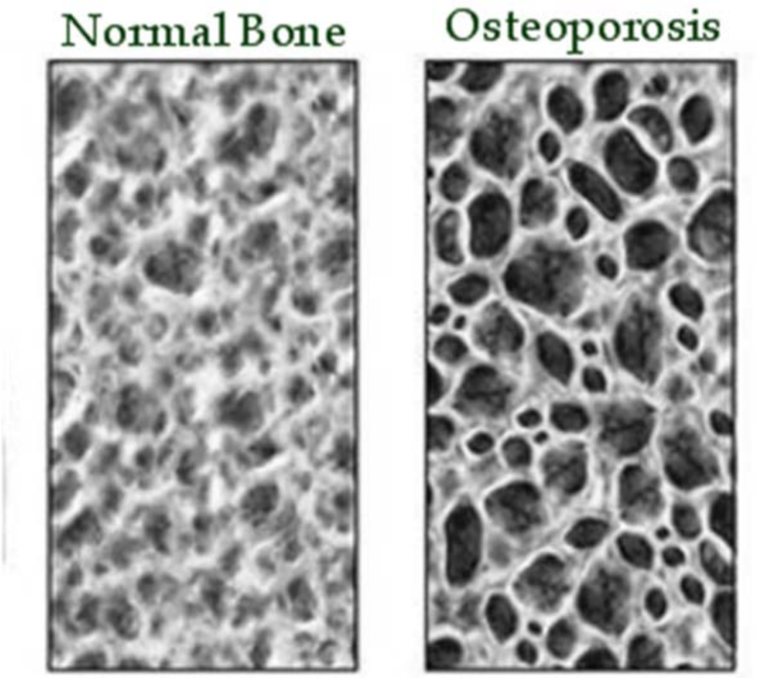
5. Calculate the ratio $\dfrac{Exponent\ \ of\ \ Num.\ of\ \ rulers\ \ in\ \ base\ 10}{Exponent\ \ of\ \ ruler\ \ lenght\ \ in\ \ base\ 10}$

$\dfrac{\quad}{-0{,}30} \approx$ $\qquad \dfrac{\quad}{\quad} \approx$ $\qquad \dfrac{\quad}{\quad} \approx$ $\qquad \dfrac{\quad}{\quad} \approx$

6. What is the fractal dimension of *Estany de Banyoles* coastline?

6-Bone cross-section

- Look at the digital images of a cross-section of bones



- Using the box counting method it is known that cross-section of normal bones have fractal dimension between 1.7 and 1.8.

- Osteoporosis is a condition of decreased bone mass. This leads to fragile bones which are at an increased risk for fractures.

- Choose the word Lower or Bigger to complete the sentence:
…………….. fractal dimension is an index for the early diagnosis of osteoporosis.